DOCUMENT RESUME

ED 143 504 SE 022 972

TITLE	Guidelines for Mathematics Instruction in Indiana Schools, 1977.
INSTITUTION	Indiana State Dept. of Public Instruction, Indianapolis.
PUB DATE	77
NOTE	84p.; Replaces ED 036 439
EDRS PRICE	MF-$0.83 HC-$4.67 Plus Postage.
DESCRIPTORS	Curriculum; *Curriculum Guides; Elementary School Mathematics; Elementary Secondary Education; *Instruction; *Mathematics Education; *Objectives; Secondary School Mathematics; *State Curriculum Guides
IDENTIFIERS	*Indiana

ABSTRACT

This set of guidelines replaces the 1969 Guidelines. Planned to assist local committees in planning curricula, it contains objectives for which local school systems must determine criteria for acceptable performance. Guiding principles on philosophy, objectives, evaluation, providing for individual differences, instructional aids, and calculators are briefly discussed. Objectives for seven strands in K-8 are presented: numbers and numeration, operations and computation, geometry, measurement, problem solving and number sentences, graphing and relations, and probability and statistics. Objectives for the secondary school are presented by course. Suggestions for implementing the guide are also included. (MNS)

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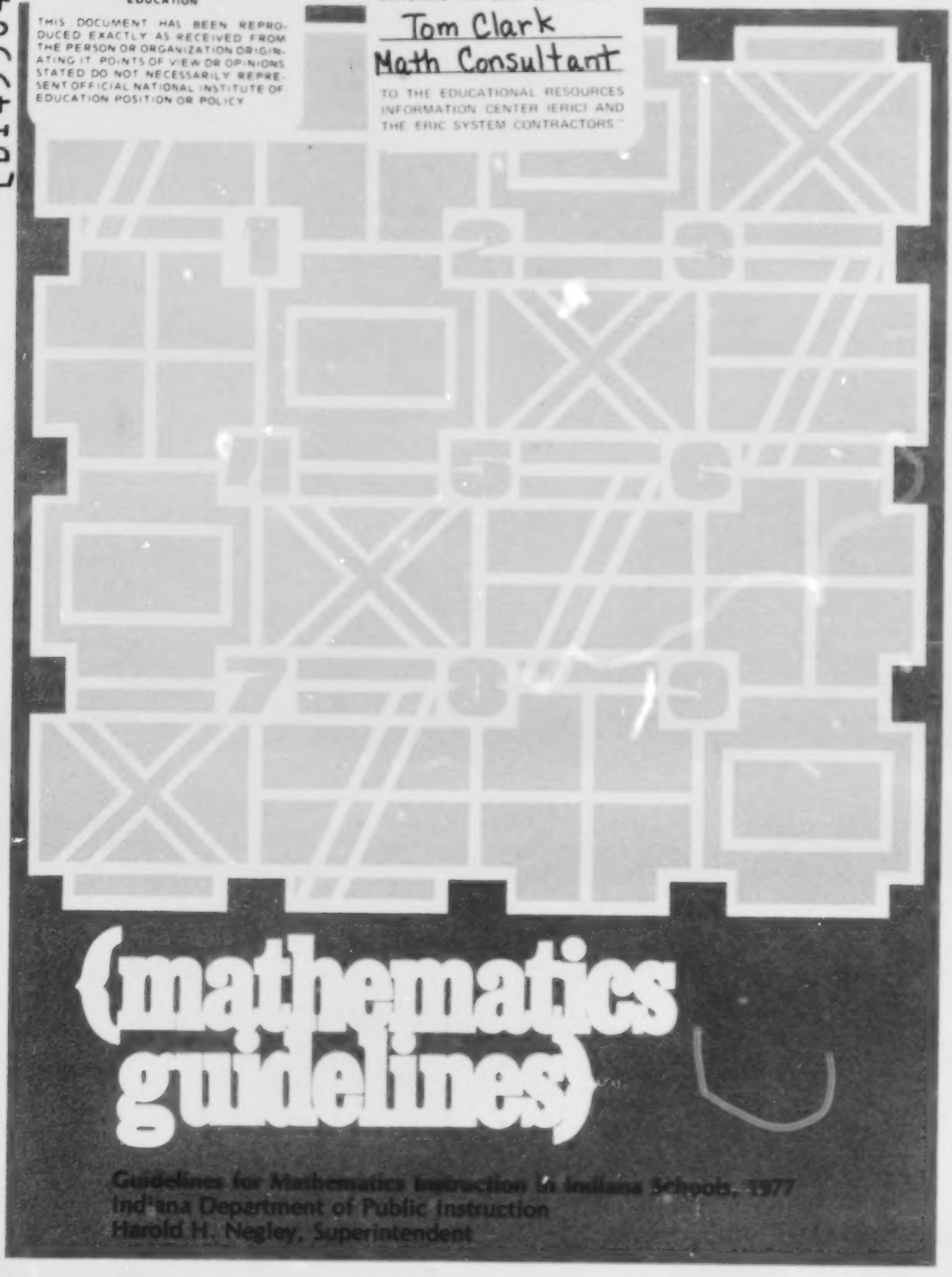

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Preface

Guidelines for Mathematics Instruction in Indiana Schools, 1977 replaces, **Mathematics Guidelines for Indiana Schools K-12**, first published in 1969. The 1977 publication was developed by the Indiana Mathematics Advisory Council under the auspices of the Superintendent, Indiana Department of Public Instruction. It is meant to be used by local educators as a guide in the evaluation and construction of their mathematics curriculum.

The initial meeting of the advisory council was held on Oct. 11, 1974. Although the agenda covered many varied subjects, the main emphasis was the development of a new state mathematics guide. At that time the council was divided into two subcommittees, one for elementary and one for secondary, to study and revise the existing guidelines. After a preliminary review by the subcommittees, individual writing assignments were made. It was then decided to continue the use of performance objectives in the new guide. However, in the development of these objectives it was determined not to include conditions of performance or criteria for acceptable performance. It was felt these items should be constructed by local school systems using the guide as a basis for the development of their local mathematics curriculum.

A preliminary draft of each section was completed in June of 1975 and submitted to all members of the advisory council for evaluation and revision. These suggestions were then given to an editorial panel, composed of six council members. The panel completed a final review and revisions in the spring of 1976.

The materials contained in this guide are not intended to replace locally developed mathematics curricula or courses of study. However, if deemed appropriate, any portion of this publication may be reproduced without permission from the Indiana Department of Public Instruction to serve such purposes. It is hoped that Indiana Schools will find this guide useful in the production of mathematics curricula and courses of study.

Superintendent's Message and Acknowledgments

This publication has been developed to aid in the coordination of mathematics education throughout Indiana schools. The primary purpose of this material is to stimulate the local school systems to instigate the evaluation or development of a complete mathematics curriculum for their students. A secondary reason is to present a set of minimal objectives that each student should achieve upon completion of a particular sequence within the total mathematics curriculum.

The Indiana Mathematics Advisory Council, composed of teachers from all levels — elementary, middle school/junior high school, high school and university as well as mathematics supervisors — have been responsible for the production of this guide. Many of the council members have previous experience in the development and writing of major curriculum projects, textbooks and local mathematics curriculum. All have a broad background of experience in mathematics education at various levels.

As previously stated, it was the council's intention that this guide should assist local committees in their endeavor to provide better and more relevant courses in mathematics. It is not the guide's purpose to be diagnostic or prescriptive. However, it is hoped that it will serve as a catalyst for local curriculum committees to develop comprehensive mathematics programs within the school system.

Many hours of thought and effort have been expended by the council toward the development of this guide. The Office of the Superintendent, Indiana Department of Public Instruction, appreciates the council's work in behalf of Indiana students and at the same time thanks superintendents of local school corporations and deans of the various universities for releasing professional personnel to complete this important document.

Substantial credit also goes to John A. Harrold, director of the DPI's Division of Curriculum, J. Thomas Pagan, publications coordinator, and Richard E. Wiley, advisory council chairman, who at the time the guide was written, was the division's mathematics consultant.

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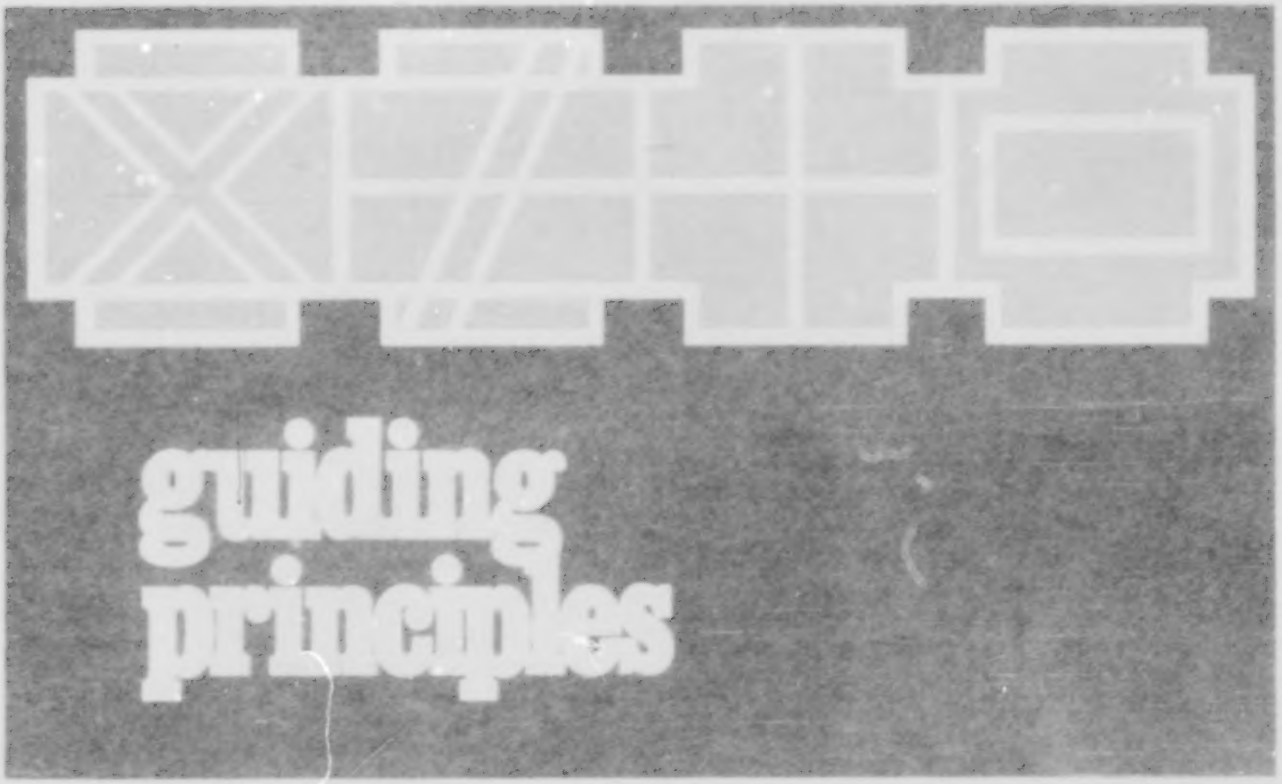


Philosophy of Mathematics Instruction

Elementary

Mathematics permeates all sectors of life. As civilization advances in technology, the focal point of communication increasingly becomes the language of mathematics. Practical situations and economic needs are factors which determine the amount and kind of mathematics to be taught. The school must assume the responsibility of giving students the mathematical skills and information which they need as children and as adults to meet the situations which require such skills and information. Meaningful mathematical experiences should lead students toward the competence and independent thought necessary for living in today's complex society.

Mathematics is a way of thinking, recording, comparing, evaluating, and measuring quantitative relationships. When it is considered only as a collection of skills, it becomes simply abstract computation and is little more than meaningless manipulation of symbols. The value and meaning of arithmetic grows out of the life situations in which number plays a part, as well as the interrelatedness of the number system and mathematical operations.

Mathematics still is directed largely to the development of certain skills and knowledge regarding numbers and their uses although marked differences have taken place in how it is taught. Today's schools have moved from teaching mathematics simply as a skill for its own sake to the need for number in understanding our world. Emphasis is given to the development of meaning and understanding as a foundation for the mechanical manipulation of numbers. Many firsthand experiences and concrete materials are used to provide the child a basis for understanding numbers and explaining arithmetical processes.

In recent years there have been some distinct trends noticeable in elementary school mathematics programs. Certain topics have been modified in order to increase relevance to business and social life thereby making number experiences more meaningful and practical for the students. The grade placement of certain topics has been raised or lowered. There has been a trend toward the introduction of more informational materials as a basis for making decisions requiring the use of numbers. This trend has resulted in the inclusion of much material which formerly was not regarded as a part of the mathematics program. Quantitative thinking does not develop merely from manipulation of numbers. Another trend has been toward emphasizing certain basic concepts such as one-to-one correspondence, number sentences, problem-solving, etc. This trend is related to the emphasis upon the early development of correct mathematical concepts, simple yet fundamental.

Modern procedures emphasize the desirability of developing a mathematics program that is in harmony

with recognized principles of child growth and development. Instead of the traditional curriculum that is organized on a compartmentalized subject basis, which stresses only the arithmetic or computational phase of mathematics, learning activities are organized in the modern program as experiences in dealing with aspects of real life situations. The teacher should be aware that learning mathematics is a gradual, developmental process. Concrete experiences to develop understanding should precede emphasis on speed and mastery.

These principles should be of help in planning and guiding number experiences:

1. Mathematics is a system of ideas, relationships and processes to be utilized in quantitative thinking.
2. Selection of content and experiences should be determined so as to develop meanings and skills which have significance in the life of the individual learner.
3. Meaning and understanding must be developed as a basis for learning processes.
4. Learning mathematics is a gradual growth process dependent upon the previous experiences and maturity of the learner.
5. Intensive, systematic, planned practice to develop and maintain proficiency is needed. However, drill alone does not necessarily produce understanding.
6. Understanding of the various ideas, processes and concepts develops gradually over a number of years.
7. Problem-solving needs to be developed in each grade through the use of many school situations involving quantitative thinking.
8. A maintenance program is especially necessary after computational skills have been understood and mastered.
9. It is recognized that learning occurs by developmental levels rather than by grade levels. Provision for difference in rates and ways of learning should be emphasized.
10. More attention should be devoted to individual differences among students in mathematics.
11. More and better types of aids for teaching mathematics should be acquired and used.
12. The mathematics program should be developed so that students get a foundation for higher mathematics and for use in everyday living. The program should be focused on the development of understanding as well as the memorization of number facts.

The search for more effective methods of teaching mathematics, combined with the needs of the future, has exerted extreme pressure on the mathematics program in the elementary school. The students need to understand the essentials of mathematics — the number system and its operations, the ideas of quantity and relationships, and the applications of concepts and theories.

The practical aspect of mathematics has been recognized and is being advanced by many. An effective mathematics program includes understanding of mathematical concepts, competency in computational skills, and instruction in their application. In a good curriculum, students are guided through a series of well-defined experiences, logically organized in terms of their ability, maturity and interests. However, the program must be influenced by other factors such as how children learn, the logical and sequential nature of mathematics, and the desire of the general public for the development of skills and competencies of a higher order.

Middle School — Junior High School

Perhaps the one factor most essential to the success of the mathematics curriculum is an emphasis on understanding. It is no longer deemed sufficient for a teacher to be satisfied with competent computational performance by a child in spite of the obvious necessity for such skill. The need to find more efficient, more enjoyable, more illuminating methods of instruction has led to the consensus that clear and penetrating understanding of certain essential mathematics should precede, but not supplant, computational skills. It is agreed that one should traverse from the known to the unknown; from the particular to the general; from the concrete to the abstract.

Students should come to realize that mathematics is a logical system which exists as a human invention, formulated, enriched, extended, and revised in response to the twin needs to develop and perfect it as a logical structure, and use it as a convenient language for describing certain aspects of nature as seen by man.

Since all students do not attain the same levels of understanding and skill simultaneously, any curriculum should possess three fundamental properties:

1. **Flexibility**
   a. to provide opportunities for students with minimum concepts to extend their basic knowledge without gaps.
   b. to provide ample stimulation and challenge for the gifted and talented to extend and enrich their learning experiences.
   c. to adjust to the rapid changes in a demanding and progressive world in which they live.
2. **Continuity**
   a. to eliminate overlapping of ideas while strengthening existing weaknesses.
   b. to fill in basic concepts and skills.
   c. to extend the knowledge and skills of all students to the utmost by a uniformly progressive development.
3. **Correlation**
   a. to integrate arithmetical, algebraic and geometric concepts and applications.
   b. to integrate mathematical ideas with various curricula in other fields through application.

With these basic ideas it becomes evident that no all

inclusive, permanent guide can be written. It is obvious that any set of objectives for a specific grade level may well be too difficult for some, too easy for others. Only general concepts can reasonably be suggested. The extent of development is contingent upon two factors — student's ability and teacher's effectiveness. Today's lesson, success or failure, should be a guide for tomorrow's materials and procedures.

Secondary

Mathematics is a language, a more specialized, exact, precise, and refined form of language than any of those to which the term "language" is ordinarily applied. It has become a powerful and indispensable tool in the description, analysis, and control of phenomena that can neither be accurately expressed nor understood in ordinary language alone. It is the most truly international of all languages. It is one of the most remarkable of all human achievements and as such forms part of the cultural heritage that schools seek to transmit and on which they build for the future.

The great majority of people are unaware of both the vastness and the diversity of mathematical thought and application. They do not realize that mathematics is the central agent for quantitative understanding of the environment; that it is in command at all points in man's efforts to apply nature to his advantage; and that is the backbone not of one science or several, but of all sciences.

Mathematics is not a set of unrelated and isolated topics. It is a system of ideas unified by such concepts as number, measurement, relationship, operation, symbolism and proof which relate to other fields of learning and which grow in meaning and significance for the student as his study continues. To nourish the growth of these concepts, to develop their relation to the problem-solving process, and to open avenues of investigation will provide the academic and vocational flexibility so essential in guiding the education of students.

Secondary school mathematics extends beyond a concern with manipulative procedures. The curriculum must afford students opportunities for working with basic concepts of arithmetic, algebra and geometry. But more than this, it must provide for the construction of symbolic and abstract models. In summary, student discovery of the internal structure and logical cohesion of mathematical systems is an important new dimension in the mathematics curriculum. The goal in every school curriculum should be to provide opportunities for four years of valuable mathematics experiences for students of all interests and abilities. △

Objectives in Teaching Mathematics

Similar problems occur when curriculum is constructed for various subjects. These include:

1. What is to be learned?
2. What sequential pattern is to be used?
3. What methodology and pedagogy is best used to accomplish the perceived goals?
4. What evaluation can be used to assess a learner's level of achievement?

This guide is an attempt to describe possible solutions to some of these questions. The two items that receive major emphasis are:

Topics which are to be learned and the sequence in which they are to be considered. To a lesser extent, a discussion of *evaluation* of a learner's achievement will be incorporated in the guide. This guide does not specify instruction techniques because the style or method of presentation depends upon the teacher, the students, the environment and available resources. As literature concerning varying methods of instruction to attain a particular outcome accumulates, it is important that teachers remain abreast of these developments.

Hopefully the day has past in mathematics education when the teacher opened the book on the first day of school and started teaching "about mathematics". Every day requires the teacher's consideration of not only what has been learned by pupils in each class, but also, what types of performance or behavior indicate that learning has taken place. In other words, consideration must be given to objectives in terms of performance. After this has been determined, the methods of instruction may be chosen from a spectrum of alternatives to elicit desired performance.

Therefore, the purpose of this guide is to stimulate teachers and curriculum developers to concentrate on learner behavior or performance that indicates learning has occurred. The next step would be to consider the means of eliciting those performances through carefully developed learning experiences.

In preparing this guide it is recognized that there exists controversy over terminology and definitions of terms relating to a statement of educational objectives. A lengthy discussion concerning the acceptability of a particular system would not be purposeful. Therefore, it has been arbitrarily decided to use the terms *global objectives, performance objectives,* and *instructional or enabling objectives.*

Global Objectives

It is recognized that mathematics education may (and perhaps should) exceed any set of stated global objectives. While it is essential that students develop basic skills and problem-solving techniques, and that they increase their mathematical literacy and their concept of the logical structure of mathematical systems; mathematics education must do more. It should stimulate a spirit of inquiry and curiosity which will enable the student to learn new mathematics by independent study. It must relate mathematics to the large body of knowledge in other disciplines. Finally, it

should consider the uses for mathematics, mathematics curriculum content areas, and abilities or behavior conducive to the effective utilization and understanding of mathematics.

With these concepts in mind one partial listing of global objectives follows:

1. An understanding of and proficiency in the mathematics that is needed for personal living and effective citizenship in our society.
2. An understanding of and proficiency in the mathematics which is necessary for various skilled jobs and professions beyond that which is needed for personal living and effective citizenship.
3. An understanding of and proficiency in the formally structured mathematics which has assumed increasing importance in the curriculum for kindergarten through college. While not obvious in the day-to-day activities of the ordinary citizen, academic mathematics plays an increasingly important role in shaping the events that affect every human being.
4. An appreciation of mathematics in the world around us.

Performance Objectives

A performance objective should have the following general qualities:

1. The learner and the instructor share a common understanding as to what is to be learned.
2. The learner and the instructor share common and firm criteria by which a learner's progress and level of competency may be assessed at specific times.

In designing this guide particular attention was given to the first of the two criteria while the second is the responsibility of the classroom teacher within the framework of the local school corporation policy. This guide includes sets of performance objectives by strands (for the students through the eighth grade) and by course (for the grades nine through twelve).

Instructional or Enabling Objectives

The term instructional or enabling objective is assigned to one or more objectives which learners must achieve before they can perform a given performance objective. These objectives do specify the conditions and criteria for acceptable performance. It is further understood that these objectives are what local teachers and curriculum developers should construct for their own curriculum guides and course offerings from the performance objectives suggested in this guide. Each performance objective listed may result in a variety of enabling objectives which are required to achieve the behavior specified in the performance objective. The relationship between the various objectives described in this section is indicated by the example that follows:

Global Objective. An understanding of and proficiency in the mathematics that is needed for personal living and effective citizenship in our society.

Performance Objective. The student should be able to perform the calculations necessary for paying taxes, e.g. sales tax, federal and state income taxes, real property taxes, personal property taxes, excise taxes, licenses, etc.

Enabling or Instructional Objective. After instruction and experience in solving various number sentences involving percents, the student will be able to compute the percentage, correct to the nearest hundredth, at least eight out of 10 times when given the base and the rate.

It will be noted that as the level of objective proceeds from global to enabling or instructional, the objective becomes more specific. This guide will be useful to local curriculum planners and classroom teachers only if they use the terminal objectives listed to develop specific instructional objectives that can be applied in an orderly fashion in the instruction of mathematics.

For a detailed procedure that can be used to develop local objectives refer to "A Mathematics Curriculum Development Process", pages 74 through 75 in the implementation section of this guide. △

Evaluation

Evaluation is an important part of the instructional program in school mathematics. Unlike its traditional role, evaluation should be more than a terminal judgment of a student's work for grading purposes. Its role should be to determine how well the basis of the instructional program, the goals and objectives, are being and have been achieved by the student and to identify difficulties which may be encountered by the student in his learning processes.

Evaluation, to be effective, must be a continual process. It should be used not only as a terminal activity but also should be used throughout the student's learning experience. It should be used to determine whether or not a planned instructional strategy is working or has been effective. It also should be used to identify the nature of a student's difficulty so that remediation may be implemented.

Evaluation probably is most often associated with the assessment of student achievement. However, there are other areas which also need to be evaluated. Evaluation should also be concerned with student attitudes, teacher effectiveness, and program effectiveness.

In the area of pupil achievement, it is true that the goal of every classroom teacher is to take a student as far as possible through the sequence of learning experiences in mathematics with understanding. In order to accomplish this goal, the teacher must first determine the knowledge and skills which the student possesses before he begins the instructional sequence. This information will enable the teacher to plan a program which will best meet the needs of the student.

During the implementation of this planned instructional program, the teacher must continually evaluate the effectiveness of the program and also must determine, at the end of the program, exactly what the student has learned.

In order to perform the various aspects of the evaluation process previously described, the teacher must use various techniques to aid in his evaluation. Some of the more useful techniques are:

1. *Classroom Observation and Interaction.* Observation is extremely useful because it provides immediate feedback for both the teacher and the student. This technique is relatively easy to apply and can provide a great deal of information in a relatively short period of time.
2. *Homework Analysis.* This technique allows for relatively immediate feedback. It permits a student to make a mistake without fear of a lower grade. This is especially important, for if students are to develop powers of inquiry and discovery they must be allowed to work in non-threatening situations where they can try different ideas.
3. *Testing.* Testing is probably the most common technique of evaluation used by teachers. Tests have the advantage that they can give the teacher a great deal of information about a student's work in a relatively short time. However, immediate feedback is not usually available to the teacher and student. A test also has the disadvantage that it is at best one sample of a student's work, and its results should be considered by the teacher in that light.

Tests are generally of two types, standardized and teacher-made. There are advantages and disadvantages to each type.

Standardized tests usually are well-written, have high validity and reliability. The use of a standardized test, however, should be considered *only* when the goals and objectives being measured by it are consistent with the goals and objectives of the instructional program. Unfortunately, this consideration is often overlooked when the decision to use, and the choice of a standardized test is made.

On the other hand, teacher made tests probably will be consistent with the goals and objectives of the instructional program. However, the test items may be poorly constructed causing low validity and reliability. It is not easy to construct a good test item, but time spent in so doing will result in more accurate evaluation. Rules for writing good test items may be found in most books on tests and measurement.

Criterion referenced tests, a recent development, are tests which are based upon a known and specified set of criteria usually stated in the form of objectives. These criteria describe performance that is to be used as evidence that a particular concept has been learned or a skill mastered. The test is matched with the objectives so that desired learning is adequately sampled.

Tests used to measure a student's achievement are not appropriate for use in diagnosis. Achievement tests will only identify *general* areas of student difficulty; diagnostic tests then should be used to identify the *specific* troublesome area.

Another important area of evaluation is that of student attitude. While attitude does not correlate with achievement, it is safe to say that a negative attitude may cause students to achieve at a lower level than the level at which they are capable of working. A student's attitude towards mathematics often may be observed directly by the teacher. This information then may be used by the teacher, if necessary, to help students change their attitude to a more positive one.

Attitude scales or open-ended questionnaires concerning how a student feels about mathematics may also be used. It must be remembered that the responses to the scale and questionnaires must be judged in light of what the teacher knows about the student from personal interaction so that the results may be judged appropriate or inappropriate.

A complete evaluation program must give attention to the effectiveness of the teacher as well as the effectiveness of the instructional program. Assessment in several areas should be considered:

1. Teacher effectiveness
2. Program effectiveness
3. Student achievement
4. Student attitude toward mathematics

In order to evaluate teacher and program effectiveness the teacher may ask questions such as:

1. Did the teacher and program provide for individual differences?
2. Did the teacher and program provide for each student's social development?
3. Were the goals and the objectives of the instructional program appropriate for each student?
4. Did the evaluation program provide a valid measure of the instructional program?

Schools may find this publication helpful in preparing measuring instruments. However, the objectives listed in this publication have not included the qualifying features, i.e., the conditions of performance and criteria of acceptable performance. It was felt by the advisory council that it would be inappropriate for the state to suggest qualifying conditions for student performance. School staffs, together with students and parents, are in an excellent position to determine more extensive objectives which will include these qualifying features. Evaluation will then become more meaningful through shared efforts and shared responsibilities. △

Providing for Individual Differences

The authors of this guide recognize the importance of providing for differences in intelligence, achievement, maturity, learning rate, emotional and social behavior, physical condition and interest throughout the student's school career. Because of this recognition, the elementary section has been prepared in terms of mathematical strands with related performance objectives, and the high school section in terms of

performance objectives for each course, along with a sequence chart and the option of providing mini-courses as needed. This plan is intended to present a working reference source on how both elementary and secondary teachers can provide an exciting, attractive learning situation for each student.

What is essential to the provision for individual differences is a firm commitment by mathematics teachers, their administrators, and parents to the idea of implementing a truly personalized instructional program in mathematics. This will enable a larger percentage of students to have successful learning experiences — successful in the sense that they have achieved identified goals. Providing a program of this kind must be carefully planned and must first consider the students involved and secondly promote a never-ending effort to make available appropriate instructional materials and equipment. The authors of this guide believe that a sensitive teacher, attuned to the emotional and intellectual needs of students, and dedicated to their individual educational welfare can help them have successful learning experiences. Further, many of these successes will occur within the structure of the traditional classroom setting if teachers and students plan their experiences to take maximum advantage of the many instructional materials and aids available to schools. Although it is not essential, many organizational variations might be utilized to help teachers provide for student differences, such as: nongraded schools; flexible, modular scheduling; team teaching; establishment of mathematics resource centers manned by the mathematics teachers; large group/small group scheduling, etc.

There are many ways of providing for individual differences; however, these should be considered valid only as the presence and extent of individual differences are determined. *It should be recognized that students at all levels of learning need to be offered variations of instruction. The method of subjecting students to the constant loneliness of working on instructional packets, day in and day out, could prove very harmful. This method is not individualization, but solitary confinement.* It should also be recognized that groupings of students are valid for some topics of mathematics instruction, while completely irrelevant for others.

Teachers can provide for individual differences in teaching certain topics in their classrooms. The following techniques are successfully being practiced in many classrooms:

1. Grouping students within the classroom according to their level of mathematical development and presenting to each group subject matter which fits that group in its scope and/or depth.
2. Grouping students from several classrooms for large group presentations by teachers cooperating and utilizing each other's strengths.
3. Grouping students by teams of two or more in which the leadership role is assumed by the students under the teacher's guidance.
4. Making flexible and challenging assignments which are both real and practical to the student's life experiences.
5. Providing a special setting for the study of mathematics where experienced aid from teachers or talented students is readily available.
6. Providing a well-equipped mathematics laboratory which will supply appropriate alternate materials such as: a) programmed materials, b) prepared tapes, c) film loops and motion pictures, d) games and activities, e) calculators, f) abacus, g) problems to solve from other academic and vocational areas.

Other suggested activities for differentiating instruction and caring for individual differences are:

1. Individual and group reports on topics investigated, making use of mathematics books available in the school library for the general student as well as the specialized student.
2. Application of mathematics in solving school and community problems.
3. Use of supplementary textbooks.
4. Mathematics clubs.
5. Mathematics exhibits and contests.
6. Constructing models and other visual aids.
7. Activities which promote parent-student interaction.

The following steps are offered for use in your efforts to individualize instruction:

1. Specify objectives in terms of student behaviors.
2. Select or devise instruments and procedures to measure attainment of objectives.
3. Pretest to determine which objectives the student has already mastered.
4. Diagnose each student's characteristics as a learner in relation to specific skills or concepts.
5. Prescribe for each student learning activities that will lead to mastery of these skills or concepts.
6. Select learning materials and equipment the student will require.
7. Program oneself as teacher to guide the student in terms of his/her prescription.
8. Personalize instructional activities and revise the prescriptions as needed.
9. Keep adequate records of pupil performance and progress.
10. Keep lines of communication between students and parents wide open.

Using a well-devised plan of personalized instruction will help teachers of mathematics in grades K-12 provide students an informational, inspirational, enjoyable and worthwhile learning experience throughout their school careers. △

Instructional Aids

Instruction in mathematics, as in all subjects, can be made more effective through the proper use of suitable teaching aids. Although chalk and chalkboard are still necessary and important, they alone are no longer considered adequate for effective mathematical instruction. Mathematics is primarily concerned with ideas and abstractions, but an understanding of the ideas often grows from concrete experiences. Teachers who keep abreast of innovative instructional materials and use them properly to enhance their instruction (when these materials have been proved to have merit) are most likely to be more effective in reaching a greater number of students.

Research by learning theorists supports the idea that instruction in mathematical principles develops progressively from active, concrete experiences to increasingly complex mathematical symbolism. Active learning, student involvement, student participation and relevance are educational concepts that receive the support of present day educators. Concerned instructors are trying to teach students to relate mathematical ideas and symbols to real objects to bring a sense of realism. Math resource centers, learning centers or math laboratory rooms are all examples of this approach to learning mathematics.

There is a great need for teachers to use *discrimination* in choosing from the wealth of aids that are being marketed. Catalogs filled with offerings are available, but many teachers and pupils find it more economical to make their own simple devices and save money for larger, more complicated devices. Keeping a "want list" is sometimes advisable in order to be ready with good suggestions for major purchases. Nonconsumable items can sometimes be purchased with federal title funds.

Instructional aids may be classified as follows:

1. Major equipment: projectors (16mm film, 8mm loop, overhead, filmstrip [classroom or small group], opaque), slide screens (small and large), tape recorders, television receivers, record players, computer terminals, individual computers, laminating and dry mount machines.
2. Devices or models: metric equipment, geoboards, games and puzzles, attribute cards, geometric figures, scales, clocks, thermometers.
3. Audio-visual materials: films, film loops, filmstrips, transparencies, acetate sheets, blank tapes.

Instructional aids contribute to effective teaching techniques in many ways. The following uses are cited.

1. Relate learning to past experiences and provide new experiences when needed.
2. Provide challenging problems for students to investigate and solve.
3. Allow students to take more responsibility for their own learning by: a) generating individual or class activities utilizing the school and local environments. b) choosing from a set of options to reinforce or review skills and concepts. c) engaging in other open-ended activities.
4. Help students use discovery approach for concepts and principles.
5. Increase motivation.
6. Provide variety in practice and drill.
7. Provide visual or tactual reinforcement of ideas.

Teachers who wish to keep current on the latest developments in the field of mathematics teaching aids are encouraged to attend meetings of the Indiana Council of Teachers of Mathematics and the National Council of Teachers of Mathematics and other professional organizations. The exhibit areas at such meetings provide a display of nearly all materials and devices that are available. Curriculum resource centers should be utilized when available. The teacher will find it is more enlightening and satisfying to examine these aids personally than to look at displays in catalogs. (There is evidence that aids in the classroom are more valuable than pictures and far more effective than words alone). If the teacher is unable to attend one or more of these meetings, the *National Council of Teachers of Mathematics 34th Yearbook of Instructional Aids in Mathematics* is an excellent source of ideas as well as an extensive bibliography for instructional aids. Teachers who develop and use their own aids with success should be encouraged to share them with others by writing articles for publication concerning the materials and their applications in the classroom.

A partial listing of sources of instructional materials and the names of the Indiana representatives of these firms appear each year in the back of the Indiana School Directory published by the Office of the Superintendent, Indiana Department of Public Instruction. A copy of this directory is sent to each local superintendent. Additional copies may be obtained by writing to the Division of Publications, Indiana Department of Public Instruction, Room 119, State House, Indianapolis, IN 46204. There is a $2 charge for each directory. Δ

Minicalculators in Mathematics Classes

The advent of inexpensive, yet powerful, "handheld" calculators must be considered in connection with mathematics curriculum revision. Although many questions about the potential of using calculators in mathematics classrooms remain unanswered, the National Council of Teachers of Mathematics has taken a positive stand and encouraged exploration and research. Preliminary investigations are inconclusive. Only the future can reveal the emphasis and direction that will be followed.

Calculators certainly will have an impact on mathematics curricula. They may change not only the kinds of computational skills which are taught but the manner in which they are taught. It is our feeling that mathematics teachers and curriculum planners must incorporate calculators into regular classwork rather than ignore or banish them. Teachers must find effective uses at all levels from primary grades to calculus.

Standardized achievement tests generally cover three phases of mathematics — computation, mathematical concepts and problem-solving. Minicalculators are computational wizards and children can use them for basic operations with a minimum of instruction. The unanswered question at this stage is, "How can paper and pencil computational skills be preserved if calculators are available?" The countless hours spent on computation drill during the middle grades needs to be reevaluated. Is checking of answers by calculators all that can be done in this area? Many educators believe it is possible to *improve* computational skills, for example, in estimation and place value determination, by using calculators.

The second area of standardized testing is that of mathematical concepts. Instruction in primeness, factorization, odd or even, LCM, and other phases of number theory can be enhanced with use of the calculator. The concept of limits and related theorems can be taught with greater effectiveness by using calculators. There are many other concepts which can be developed or reinforced through the use of calculators and deserve greater exploration.

Finally, standardized tests include a subtest on problem-solving, sometimes called applications. Here is a prime area where calculators should be used effectively. Too often there is little time spent on problem-solving in mathematics classes due to difficulties with computation. This is true despite the obvious importance of practical applications for the majority of students. Also, many textbook problems are trivial and the numbers are kept artificially simple because of difficult calculations involved. This need be true no longer since the minicalculator can perform realistic computations as easily as the simplified versions found in texts. The student is provided the opportunity of keeping abreast of the problem at hand without getting sidetracked with tedious computations.

The grade level and specific courses will influence potential uses of calculators. In advanced mathematics courses, the value of scientific calculators, including programmable types, is quite evident. They will replace slide rules and books of tables. We believe calculators belong in all advanced mathematics classes as invaluable tools.

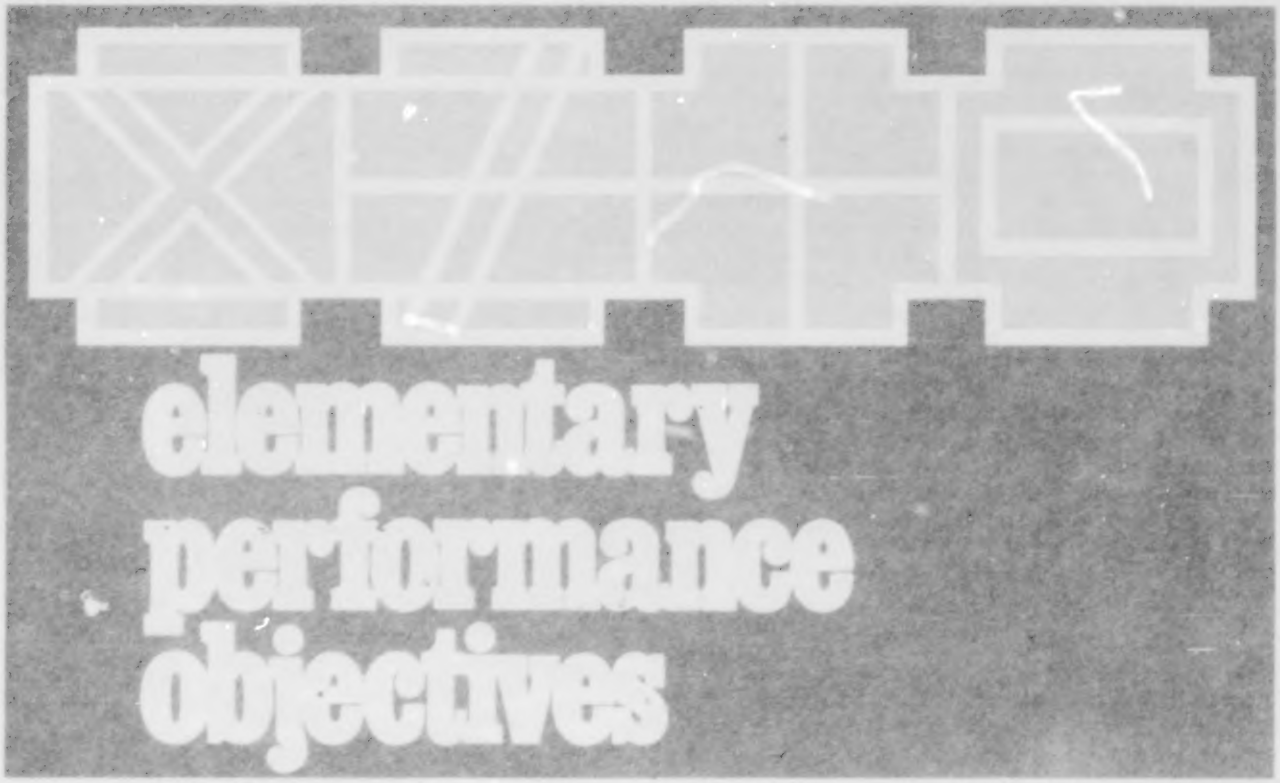
Nearly all prerequisite skills to higher mathematics can be strengthened by well-planned experiences with the calculator. Experimentation with the use of calculators in mathematics classrooms at all levels is strongly recommended.

Following are some suggested uses of calculators in the classrooms:

1. Reinforce computational skills.
2. Improve estimation of results.
3. Aid in teaching place value.
4. Develop number concepts.
5. Stimulate interest through games.
6. Solve problems with factual data e.g., local or national sports statistics, business data, consumer buying, discounts, etc.
7. Check answers to computation.
8. Drill on arithmetic "facts" (alternative to flash cards).
9. Extend problems in the text to use larger or more realistic numbers.

A goal concomitant to learning more useful mathematics is the development of skill and confidence in using the calculator. Many adults are reluctant to use calculators and school is the most effective place for such orientation and practice.

In conclusion, it is clear that calculators are becoming a regular part of daily life for many people. The mathematics classroom is the logical place to prepare future breadwinners for the increasing complexity of modern life — including effective use of calculators. Experimentation with the use of calculators in mathematics classrooms at all levels is strongly recommended. △



Mathematics by Strands

The elementary section of this guide is organized in terms of mathematical strands. These strands might be listed in many different ways, and in many instances are listed differently by other mathematics educators. The seven strands by which the elementary school program is organized in this guide are not necessarily the only strands that might have been listed. However, in the opinion of the writers of this publication, these strands are sufficiently distinct and important in developing the content of an elementary school mathematics program.

A strand may be regarded as a concept that is an integral part of the elementary program from preschool and kindergarten through grade eight. The concepts of a strand that are developed in the first grade are expanded and extended in the second grade and beyond. To provide for continuity and individual differences, there exists an overlap of objectives within each strand. In viewing a mathematics program in this manner, it may be easier to observe the organization and structure within which a child grows in mathematics knowledge. The seven strands that the authors of this guide have selected are:

(1) **Numbers and Numeration**
(2) **Operations and Computation**
(3) **Geometry**
(4) **Measurement**
(5) **Problem-Solving and Number Sentences**
(6) **Graphing and Relations**
(7) **Probability and Statistics**

These strands or variations thereof are found in the scope and sequence charts of most current elementary mathematics textbook series.

The reader will note that neither Logical Thinking nor Estimation are listed as strands and may question the reason for these items not being included. The authors of this guide see these topics as competencies to be achieved by students that are not solely within the realm of mathematical content. These competencies should be acquired by students as they study any mathematical strand contained in this guide. However, as mathematics content is being taught, teachers should be aware that students should be gaining proficiency in the use of Logical Thinking and Estimation.

The fact that these proficiencies are not listed as strands is *not* an indication that they are unimportant. On the contrary, the student's acquisition of these competencies is one thing a teacher should monitor as mathematical content progresses. The content strands should be utilized *only* as a guide in helping teachers become more aware of competencies students need to acquire and the order in which they should acquire them.

A brief description of each of the strands follows:

Numbers and Numeration. This strand deals with the study of numbers and how they are named as well as man's way of recording numbers in a systematic form. The decimal numeration system is carefully

studied. Other numeration systems are referred to for comparative properties and cultural values. This strand in elementary school mathematics deals with whole numbers, non-negative rational numbers, and integers. The strand also includes the study of prime numbers, composite numbers, and other elements of number theory.

Operations and Computation. This strand deals with the basic operations on numbers, namely: addition, subtraction, multiplication and division. All the standard algorithms are treated extensively. This strand also examines the properties of numbers under the four fundamental operations.

Geometry. This strand develops the elements of spatial relationships. It examines particular arrangements of points, lines, planes and spaces. Identification and utilization of special geometric shapes are introduced and explored. Metric geometry deals with the measures of important figures, such as triangles, rectangles and circles. Non-metric geometry deals with properties and relationships among these figures.

Measurement. This strand is devoted to the method of recording measures of measurable quantities such as time, mass (popularly called weight), temperature, length, etc. This strand overlaps with the metric geometry strand in measures of length, areas and volume. Within this strand both the metric system (SI) and the customary system should be taught as independent systems of measure.

Problem-Solving and Number Sentences. This strand involves the method of translating verbal and physical situations into mathematical language and the processes by which the solutions are found and interpreted. The relationships of equality and inequality are studied in the form of open sentences, as well as statements.

Graphing and Relations. This strand deals with the representation of numbers on a line and pairs of numbers on a coordinatized plane. This strand also deals with the relation of one set of numbers to another as depicted by various types of graphs.

Probability and Statistics. This strand focuses on the collection and interpretation of data through the use of fundamental analysis procedures. It also encompasses the areas of chance and other uses of the laws of probability. Δ

Objectives K-8 [by grade level and strands]

Performance Objectives, K-8 (by grade level and strands)

	Grade Level Objectives								
Strand	K	1	2	3	4	5	6	7	8
Numbers and Numeration	1 to 9	1 to 15	10 to 29	19 to 35	27 to 39	36 to 48	41 to 54	43 to 61	43 to 61
Operations and Computation	1 to 3	1 to 8	7 to 14	13 to 24	17 to 35	33 to 47	41 to 52	45 to 56	45 to 58
Geometry	1 to 4	1 to 8	5 to 13	7 to 19	13 to 29	19 to 43	34 to 47	37 to 55	37 to 57
Measurement	1 to 4	1 to 8	6 to 13	10 to 19	16 to 20	17 to 23	21 to 26	25 to 29	25 to 31
Problem-Solving and Number Sentences	1 to 2	1 to 9	3 to 15	10 to 19	14 to 23	16 to 28	24 to 34	24 to 37	24 to 44
Graphing and Relations	1 to 2	1 to 4	1 to 6	3 to 10	7 to 15	10 to 19	15 to 22	10 19 to to 14 30	10 19 to to 14 33
Probability and Statistics	1 to 4	1 to 5	3 to 6	5 to 9	7 to 11	9 to 15	12 to 20	16 to 24	19 to 26

Numbers and Numeration

The objectives in this strand include those concepts and skills associated with the child's understanding of number (whole, integer and non-negative rational) and the place value system used in our society. The list of objectives is not complete; for example, at the upper grade levels the concept of place value is not fully developed. However, the same process and objectives for development should be similar to those used at earlier grades. In the same spirit, many objectives are given as a guideline to incorporating the topic rather than trying to be all-inclusive in the listing. An example of this occurs in objectives associated with number theory and rational numbers in fractional form. The list of objectives, if it were all-inclusive, would be too long.

The importance of this strand is known and valued by every teacher. It forms the foundation for understanding all concepts and computation which are the core of elementary school mathematics. Individual teachers are advised to read all of the objectives in an effort to see the continuous growth that is suggested in this list. Number facts and the development of the four fundamental operations are not included in this strand but will be found in the strand titled "Operations and Computation."

Numbers and Numeration: Performance Objectives

SUGGESTED GRADE LEVELS

	Preschool and K	1	2	3	4	5	6	7	8
ITEMS	1 to 9	1 to 15	10 to 29	19 to 35	27 to 39	36 to 48	41 to 54	43 to 61	43 to 61

Objectives	Examples
1. Sort a group of objects into sets that are alike in some way.	1. Child sorts a collection of objects (blocks, buttons, beads) so that the things in each pile are alike in some way (color, shape, size).
2. Compare two objects on the basis of a common property.	2. Child is given two sticks that differ only in length and tells which one is longer.
3. Classify a set of objects on the basis of a common property.	3. Child is given a set of objects that differ only in shape and groups them by shape.
4. Compare two sets of objects and determine which set has more objects.	4. Child finds that there are more children than balls, as he passes out balls at play time.
5. Form a one-to-one correspondence between two sets of objects which have the same number of elements.	5. Child gets a book for each child in his group.
6. Tell how many in a set; one through five; zero through nine.	6. Child tells how many blocks in a box; or exhibits an appropriate set when asked to "give me three blocks."
7. Place sets of objects in order according to the number of objects in each set, from the least to the greatest.	7. Child arranges sets of blocks in order:
8. Identify the symbols 0 through 9 when the name is given orally.	8. Teacher says, "Point to the numeral that means '5'."
9. Count the number of objects in a set of 10.	9. Count through 10 placing number names, beginning with one, in one-to-one correspondence with objects being counted.

10. Read and write numerals to 10.

10. Write the numeral for the number of the following sets:

_____ _____ _____

11. Group objects in sets of 10.

11. a) Use finger plays involving 10 fingers.
b) When taking attendance match each child with a bead on the counting frame; children decide how many rows of 10 beads and how many more are needed.

12. Match the ordinal number names with an ordered set containing up to 10 elements.

12. Point to the second figure from the left:

13. Infer a relationship of greater than, equal to, or less than between two numbers after telling whether one set has as many as, fewer than, or more than another set.

13.

(? is replaced by "is greater than," "is less than," or "is equal to.")

14. Write numerals for one more, one less.

14. "One more than six is _____"; "One less than six is _____."

15. Mark fractional parts of objects; halves, thirds, fourths.

15. Color one third of the circle.

16. Use equivalent fractions concept informally in experiences.

16. 1/2 of a pie is equivalent to 2/4 of a pie.

17. Match numerals for sets, write numerals for and order sets 10-19.

17. Write numerals for the following sets:

XXXXXXXXXXXXXX _____

XXXXXXXXXXXXXXXX _____

18. Count by ones, two, fives and tens to 100.

18. "Starting at 35, count by fives to 50."

19. Give examples which relate experience with negative numbers.

19. Children discuss meaning of below 0; going "in the hole" in a game.

20. Tell which numbers in a set are even, which are odd.

20. Which of the following numbers are even?

{3, 6, 9, 12, 16, 19}

21. Match numerals with sets of tens and ones up to 99.

21. Match the following:

////////// 25

////////// //// 10

////////// ////////// ///// 14

22. Illustrate the tens and ones represented by any numeral up to 99.

22. Draw a picture which shows bundles of tens and ones for 26, 31, . . . etc.

////////// ////////// //////

23. Read and write numerals for sets from 10 up to 99.

23. Name the numerals for the following:

////////// ////////// /// 23

////////// /////// 17

24. Order a set of numbers (0 to 99) from the smallest to the largest.

24. Order the set of numbers {21, 12, 5, 81, 19} from the least to greatest.

$5 < 12 < 19 < 21 < 81$.

25. Name the whole number immediately before and immediately after any number (1 to 99).

25. Name the whole number that comes immediately before and immediately after:

_____, 70, _____

_____, 19, _____

26. Tell the meaning of each digit for numerals representing numbers less than 100.

26. The children bundle 62 popsicle sticks into six bundles of 10 with two left over and read the numeral which shows how many tens and how many ones. "62" is "sixty-two" meaning "six tens and two ones."

27. Read and write any numeral up to 999.

27. "Write the numerals as I dictate them -- 74, 809, . . . Read the numerals I have written on the chalkboard."

28. Order a set of numbers (0 to 999) from least to greatest.

28. List the set of numbers {0, 795, 54, 28, 984, 231, 321} in order from the least to the greatest.

29. Explain the place value meaning for any number to 999.

29. In 234, the 2 means 2 hundreds, the 3 means 3 tens, and the 4 means 4 ones, or:

$234 = 200 + 30 + 4$

30. Rename hundreds, tens and ones (for work with operations).

30. 432 can be renamed as 4 hundreds, 2 tens, and 12 ones.

31. List a set of fractions equivalent to a given fraction.

31. List two fractions equivalent to each of the fractions: 1/4, 1/3 and 5/2.

$\frac{1}{4}$ $\frac{2}{8}$ $\frac{3}{12}$

$\frac{1}{3}$ $\frac{2}{6}$ $\frac{3}{9}$

$\frac{5}{2}$ $\frac{10}{4}$ $\frac{15}{6}$

32. Order a set of rational numbers.

32. Put the set of numbers {3/4, 2/3, 1/4, 1/3, 1/2} in order from least to greatest.

33. List the first eleven multiples of 2, 3, 4, 5, 6, 7, 8 and 9.	33. Set of multiples for 6: $\{0, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60\}$ (Note: 0 is a multiple of every whole number.)
34. Predict "evenness" or "oddness" of the sum, difference or product of two numbers.	34. Will the sum of 17 + 27 (two odd numbers) be even or odd?
35. Express place value concepts to 999,999 (development of these concepts parallels earlier development.)	35. 99,999 means 9 ones, 9 tens, 9 hundreds, 9 thousands, 9 ten-thousands.
36. Find the common factors of two natural numbers.	36. The common factors of 10 and 12 are 1 and 2. The common factors of 6 and 30 are 1, 2, 3, and 6.
37. List the prime numbers that exist in some interval.	37. The prime numbers between 30 and 50 are 31, 37, 41, 43, and 47.
38. Determine the prime factorization for a composite number no greater than 40.	38. $36 = 2 \times 2 \times 3 \times 3$
39. Write numerals for a set of fractions equivalent to any given fraction.	39. Write the numerals for three other fractions equivalent to 1/3. The set of fractions equivalent to 1/3 is $\{1/3, 2/6, 3/9, \ldots\}$.
40. Give examples which relate positive and negative numbers to realistic situations.	40. Pairs of numbers relate to temperature changes, and differences between positions or scores.
41. List a given set of positive rational numbers in order from the smallest to the largest.	41. 1/6 2/5 1/2 4/7 8/9 10/9 $.003 < .004 < .02 < .1 < 1.2$
42. Express any composite number as the product of primes.	42. 60 expressed as the product of primes is $2 \times 2 \times 3 \times 5$.
43. Read and write whole numbers up to 1,000,000,000 (one billion) and in expanded form up to one million.	43. Read 709,843,956,154 Write two million, three hundred, etc. $456 = 4\ (10)^2 + 5\ (10) + 6$
44. Write decimal numbers to thousandths.	44. .7 .81 .236
45. Write fractions in simplest form.	45. 4/6 = 2/3 or 10/4 = 5/2
46. Write terminating decimals as fractions.	46. .2 = 1/5 0.125 = 1/8
47. Write some fractions as terminating decimals.	47. 4/5 = .80 3/4 = .75
48. Write mixed numerals in fraction and decimal form and conversely.	48. a) 3 1/2 = 7/2 = 3.5 b) 2.75 = 2 3/4 = 11/4
49. Round numbers as required.	49. a) round to nearest tenth 3.184 to 3.2 b) round to nearest hundred 572 to 600 c) round to nearest thousandth .0874 to .087
50. Find the least common multiple of two natural numbers.	50. The least common multiple of 6 and 9 is 18.

51. Find the greatest common factor of two natural numbers.

51. The greatest common factor of 30 and 40 is 10.

52. Write fractions as either terminating or repeating decimals.

52. $1/8 = .125$, $2/11 = .\overline{18}$, $2/7 = .\overline{285714}$

53. Represent a given positive rational number using decimal, common, mixed-numeral, percent and scientific notation.

53.

3/5	.6	60%
2.35	235/100	235%
35%	35/100	.35
7/5	1.4 140%	1 2/5
	1.4×10^0	

54. Illustrate the union and intersection of any two sets with Venn diagrams.

54. In the diagram below, Set A is represented by the circle on the left and Set B by the circle on the right. Shade in the region which represents the intersection of the two sets.

55. Order any set of integers from the least to the greatest.

55. Order this set of integers from least to greatest. $\{1, -5, 6, 2, -19, 28\}$

56. Distinguish between a rational and an irrational number.

56. .1010010001. . .; $\sqrt{2}$; π (irrational)

.175; $1.\overline{731}$; $\sqrt{9}$ (rational)

57. Find a rational approximation for an irrational number such as $\sqrt{2}$, $\sqrt{3}$, etc. to the nearest hundredth. (By using a table or calculator).

57. $\sqrt{2}$ is approximately 1.41

$\sqrt{3}$ is approximately 1.73

58. Identify the additive and multiplicative inverses for whole numbers, integers and rationals (if possible).

58 Which of the following have additive and multiplicative inverses. Give examples.

Whole numbers 2, 8
Integers 3, -7
Rationals 3/8, -2 1/2

59. Express numbers in different bases.

59. This set of tally marks / / / / / / / / / / / is recorded as: 10 in base ten; 1010 in base two; 20 in base five.*

60. Write the base five numeral for a number whose base ten numeral is given.

60. Write the base five numeral for the set of stars.*

• • • • •
• • • 23_{five}
• • • • •

61. Find the Cartesian product of two finite sets.*

61. If $A = \{a, b, c\}$ and $B = \{1, 2\}$ then
$A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)\}$ *

*Optional

Operations and Computation

This strand is an integral part of all mathematics instruction; just as the use of operations within various number systems is essential to application of the systems in solving problems. Declines in arithmetic skills indicated by standardized tests have led to criticism of "New Math" and a resultant desire to place renewed emphasis on the teaching of the aforementioned skills.

For those who have an inclination to increase the emphasis placed on the teaching of computational skills, caution should be exercised so as to not regress to the routine drill and practice of the past. While the learning of the number facts is important, the development of understanding of mathematical concepts is perhaps more crucial.

The teaching of the metric system will have a definite effect on this strand. Since the metric measurement system is a decimal system, there will be a greater concentration placed on the use of the decimal system and a resultant decrease in the use of fractions.

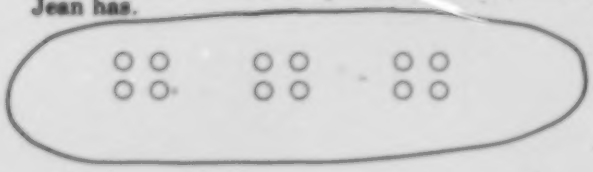
Another factor which must be considered in conjunction with this strand is the advent of calculators and their subsequent use in elementary mathematics instruction. The use of these instructional aids will require greater emphasis being placed on estimation and approximation within this strand. (See "Minicalculators in Mathematics Classes" in the general section of this guide).

Operations and Computation: Performance Objectives

SUGGESTED GRADE LEVELS

	Preschool and K	1	2	3	4	5	6	7	8
ITEMS	1 to 3	1 to 8	7 to 14	13 to 24	17 to 35	33 to 47	41 to 52	45 to 56	45 to 58

Objectives	Examples
1. Combine two sets into a single set (introduction to addition).	1. Use bottle caps to form sets that look like and Then combine the two sets. Demonstrate first with physical objects, then draw a picture to illustrate. (The ability to count is not necessary here).
2. Separate some members (subset from a set) (introduction to subtraction).	2. From "take away" and show the set which remains. Demonstrate first with physical objects and then illustrate with a drawing. (The ability to count is not necessary here).
3. Demonstrate a set which when combined with a given set produces another given set.	3. Produce the set which when combined with gives Answer

4. Illustrate, recall and use addition facts up to sums of 18.	4. Introduction and basic use of the addition facts using counters or other aids to show, for example: $5 + 8 = 13$
5. Demonstrate the sum of two addends is not affected by their order.	5. Use counters to show that $4 + 3 = 3 + 4$.
6. Illustrate, recall and use the subtraction facts based on sums up to 18.	6. Introduction and use of the basic subtraction facts using counters or other aids to show, for example: $11 - 7 = 4$
7. Show the relationship between addition and subtraction.	7. Given three numbers such as 3, 4, 7, the related number sentences such as $3 + 4 = 7$ and $7 - 4 = 3$ should be demonstrated.
8. Demonstrate that the sum of three or more addends is not affected by their grouping.	8. Illustrate using counters that $2 + (7 + 3) = (2 + 7) + 3$.
9. Using the addition facts to add numbers through hundreds without regrouping.	9. Add without regrouping. a) $\begin{array}{r} 34 \\ +\ 5 \\ \hline \end{array}$ b) $\begin{array}{r} 342 \\ +546 \\ \hline \end{array}$
10. Use the subtraction facts to subtract numbers without regrouping.	10. Subtract without regrouping $\begin{array}{r} 645 \\ -234 \\ \hline \end{array}$
11. Give the relationship between addition and subtraction.	11. Write the two addition sentences using the numbers (8, 7, 15). Write the associated subtraction sentences for each addition sentence.
12. Use the order (commutative) and grouping (associative) properties in addition.	12. The student should be able to illustrate using counters that $4 + 7$ is the same as $4 + (2 + 5)$.
13. Recognize a simple multiplication situation.	13. Jean has three sets of dishes. Each set has four dishes. Draw a diagram showing how many dishes Jean has. 
14. Add two or more numbers which are each less than one thousand by using regrouping when necessary.	14. $\begin{array}{r} 852 \\ +586 \\ \hline 1438 \end{array}$
15. Apply zero as the identity element in addition.	15. What number added to 6 will give a sum of 6? Answer: zero (0). Students should be able to write the number sentences $0 + 6 = 6$ and $6 + 0 = 6$.
16. Find the difference of two numbers each less than one thousand using regrouping when necessary.	16. $\begin{array}{r} 738 \\ -457 \\ \hline 281 \end{array}$
17. Estimate sums and differences prior to the actual computation.	17. a) $\begin{array}{r} 647 \\ 398 \\ +240 \\ \hline \end{array}$ the sum is less than 1400 but greater than 1200. b) $\begin{array}{r} 874 \\ -\ 98 \\ \hline \end{array}$ the difference is less than 800 but greater than 700.

18. Check addition and subtraction examples using the relationship between addition and subtraction.

18. 640 − 532 = 108 because 108 + 532 = 640

    234 + 197 = 431 because 431 − 197 = 234

19. a) Illustrate, recall and use multiplication facts where one factor is five or less.

    b) Demonstrate that the product of two factors is not affected by their order.

    c) Demonstrate that the product of three or more factors is not affected by their grouping.

19. Use counters or arrays to demonstrate that

    a) 4 x 3 = 12

    b) 4 x 5 = 5 x 4

    c) (3 x 2) x 4 = 3 x (2 x 4)

20. Find products using the distributive property.

20. 4 x 13 = 4 x (10 + 3)
    = (4 x 10) + (4 x 3)
    = 40 + 12
    = 52

21. Illustrate, recall and use division facts for factors 0 - 5.

21. 18 divided by 3 = 6

22. Show the relationship between multiplication and division.

22. Write the multiplication sentences using 2, 5, 10. Write the associated division sentence for each multiplication sentence.

23. Use "patterns" to distribute two or three digit dividends to divide.

23. 48 divided by 4 = (40 ÷ 4) + (8 ÷ 4)
    = 10 + 2 = 12

24. Extend the division algorithm to vertical form. Use division as repeated subtraction and remainders of zero.

24.
```
   12  R  0
4 ) 48
    40     10
    --     --
     8
     8      2
     -      -
     0
```

25. Find sums and differences using numbers up to one hundred thousand using regrouping when necessary.

25. a) 51216 + 9845 b) 4842 − 951

26. Choose the fraction which correctly identifies that part of the figure or set which is shaded.

26. a) Choose a fraction which represents the shaded part of the figure.

1/2, 1/3, 2/3, 2/4

b) Choose the fraction which represents the shaded part of the set.

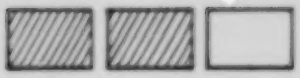
1/2, 1/3, 1/4

27. Choose the picture where that part of the figure which is shaded denotes a given fraction.

27. Choose the diagram where the shaded part represents 3/4:

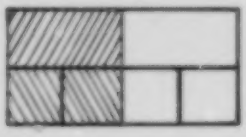
28. Write the fraction which expresses that part of the diagram which is shaded.

28. a)

b) 

29. Use figures to show that two given fractions are equivalent.

29. Use the figures below to show that 1/2 = 2/4



30. Illustrate, recall and use the multiplication facts up to 9 x 9.

30. 8 x 9 = 72

31. Identify and use the properties of zero and one in multiplication.

31. a) 0 x any number = 0

b) 1 x any given number = that given number.

32. Divide selected numbers by using the related multiplication facts.

32. a) 56 ÷ 7 = 8 because 7 x 8 = 56

b) 420 ÷ 7 = 60 because 60 x 7 = 420

33. Apply the distributive law to find the product of a two or three digit factor and a one-digit factor.

33. 9 x 81 = 9 x (80 + 1)
= (9 x 80) + (9 x 1)
= 720 + 9
= 729

34. Using numbers through thousands, use the standard multiplication algorithm.

34.
$$\begin{array}{r} 268 \\ \times\ 49 \\ \hline 2412 \\ 1072 \\ \hline 13132 \end{array}$$

35. Find the quotient using repeated subtraction and one or two digit divisors.

35.
$$\begin{array}{r} 42 \quad R3 \\ \hline 23 \overline{)\,989} \\ 920 \quad 40 \\ \hline 69 \\ 46 \quad 2 \\ \hline 3 \end{array}$$

36. Use the standard algorithm to find the quotient.

36. Divide 1987 by 42.

$$\begin{array}{r} 47 \quad R\ 13 \\ \hline 42 \overline{)\,1987} \\ 168 \\ \hline 307 \\ 294 \\ \hline 13 \end{array}$$

37. Write and order the fractions related to given diagrams.

37. Write the fractions related to each of the following.

Order the fractions.

38. Find the least common denominator (LCD) of several fractions by using the least common multiple (LCM) of the denominators.

38. To find the LCD for 1/12, 1/10, find the LCM of 12 and 10.

39. Arrange several fractions in order from smallest to largest and vice versa.

39. a) Arrange 2/3, 7/3, 5/3, 2/3 so the numbers represented go from the smallest to largest.

b) Which of 2/3 and 1/2 represents the smaller number.

40. Perform addition, subtraction, multiplication and division with fractions.

40. a) Find the sum of 3/8 and 2/8.

b) Find the difference of 1/2 and 1/3.

c) Find the product of 2/3 and 4/5.

d) Find the quotient of 3/5 and 3/4.

41. Perform multiplication and division using powers of 10.

41. $8000 \div 40 = (8 \times 10^3) \div (4 \times 10)$

$= 2 \times 10^2$

$20 \times 300 = (2 \times 10) \times (3 \times 10^2)$

$= 6 \times 10^3$

$= 6000$

42. Estimate products and quotients.

42. a) The product of 23 and 6 $>$ 120 but $<$ 180 or is between 120 and 180.

b) The quotient of 743 and 12 is $>$ 60 but $<$ 70 or is between 60 and 70.

43. Illustrate the use of the commutative, associative, and distributive properties and the properties of one and zero.

43. Complete the following open sentences.

$23 + 457 = \square + 23$

$7 \times (8 \times 5) = (7 \times \triangle\ ) \times 5$

$73 \times (\ \square\ ) = (73 \times 10) + (73 \times 7)$

$1 \times 9 = 9 \times 1 = \triangle$

$16 \times 0 = 0 \times 16 = \square$

44. Write the decimal equivalent of a fraction and vice versa.

44. a) 3/4 = .75

b) .4 = 2/5

45. Arrange several decimals in order.

45. Arrange 0.3, 0.03, 0.029 in increasing orde..

46. Perform the four fundamental operations with decimals.

46. Complete to make true sentences.

$3.4 + 5.4 = \triangle$

$43 - 6.1 = \square$

$4.8 \times 4 = \square$

$4.5 \div .9 = \triangle$

47. a) Perform the four fundamental operations on the set of non-negative rational numbers.

47. a) $3/4 + 5/6 + 2/3 = 27/12$

$4\ 1/3 - 2\ 5/6 = 1\ 1/2$

$4/5 \times 8/7 = 32/35$

$5/8 \div 2/3 = 15/16$

b) Estimate the result of the operation prior to the actual computation.

b) 3/4 + 5/6 + 2/3 The sum is greater than two but less than three.

4 1/3 — 2 5/6 The difference is greater than one but less than two.

2/3 x 4/5 The product is less than one.

5/8 ÷ 3/4 The quotient is less than one.

48 a) Compute the sum, difference, product and quotient using decimals.

48. a) 0.247 + 0.132 = △

1.946 — 0.759 = □

0.6 x 0.21 = △

5.26 divided by 0.18 = □

5.26 ÷ 0.18 = □

b) Estimate the result of the operations prior to the actual computation.

b) 2.137 + 0.5874 The sum is greater than two but less than three.

9.467 — 0.7539 The difference is greater than eight but less than nine.

0.623 x 0.21 The product is less than 0.50000.

5.26 ÷ 0.18 The quotient is less than 30 but greater than 20.

49. Order a set of rational numbers.

49. Arrange 0.73, 0.075, 7 1/2 so that the numbers which are represented are in order from smallest to largest.

50. Compute the sum, product, difference and quotient of any two whole numbers.

50. 25,665 + 39,672 = □

325 x 256 = △

3,246 — 1,738 = △

2,170 ÷ 62 = □

51. Multiply mentally by powers of 10.

51. 100 (23.5) = 2350
.01 (3.4) = .034

52. Compute the sums, products, differences and quotients of non-negative rational numbers in fractional form where the numerator and denominator are less than 50.

52. 3/8 + 5/7 = 61/56; 2/3 x 7/3 = 14/9

3/8 — 1/16 = 5/16; 5/4 ÷ 3/8 = 40/12 or 10/3

53. Compute the sum and product of integers between -20 and 20.

53. 8 + (-5) = 3
(-7) + 3 = -4
(-7) x (-7) = 49
(8) x (-6) = -48

54. Compute the sum, product, difference, and quotient of any two non-negative rational numbers in decimal notation.

54. 3.24 + 2.8 = 6.05
(.2) x (.3) = .06
8.23 — 5.42 = 2.81
.45 ÷ 1.5 = .3

55. Use open sentences to illustrate the sum, product, difference, and quotient of any two non-negative rational numbers in decimal notation.

55. 2.35 + △ = 17.55

(.25) (.04) = □

△ — .397 = 1.477

18.3 ÷ 2.54 = □

56. Compute the sum, product, difference and quotient of a pair of integers between -100 and 100.

56. $85 + (-15) = \Delta$
$10 \times (-10) = \square$
$(-80) - (-30) = \square$
$(-80) \div (-5) = \Delta$

57. Identify and illustrate the commutative, associative and distributive properties for rational numbers. Also illustrate the multiplicative and additive properties of zero and one. Illustrate the additive and multiplicative inverses.

57. $5/8 + 11/6 = 11/6 + 5/8$
$2/5\ (3/8 + 1/7) = 2/5 \times 3/8 + 2/5 \times 1/7$
$(1/2 + 1/3) + 2/3 = 1/2 + (1/3 + 2/3)$
$1/2 + 0 = 0 + 1/2 = 1/2$
$1/2 \times 0 = 0 \times 1/2 = 0$
$1 \times 3/4 = 3/4 \times 1 = 3/4$

(additive inverses)
$3/5 + -3/5 = -3/5 + 3/5 = 0$

(multiplicative inverses)
$2/3 \times 3/2 = 3/2 \times 2/3 = 1$

58. Determine the approximate square root of a number by the use of an algorithm or calculator.

58. $\sqrt{20}$ is approximately 4.47

Geometry

The strand of geometry has often been neglected in the teaching of elementary mathematics. This is regrettable because recognition of shape and form relationships are early and easy experiences for children. The modern math movement of the 1960s introduced a considerable amount of geometry into the course content of elementary mathematics. The evolution of the curriculum in the 1970s toward greater concern for practical applications has not mitigated the need for a continued emphasis on the geometry strand. Research has established that children at an early age can learn to distinguish shapes and perceive spatial relationships.

Teaching the geometry strand will require use of numerous visual aids and devices, but most of these can be made by teachers or students. Look around the classroom. You can see geometric illustrations everywhere. There are rectangles everywhere, as well as triangles, circles, squares, trapezoids, cubes and spheres. Paper folding activities can dramatize relationships and make such geometric concepts as parallelism or perpendicularity visual and relevant. With effective use of readily available materials the teaching of geometry can become one of the most vibrant and interesting aspects of mathematics.

Geometry: Performance Objectives

SUGGESTED GRADE LEVELS

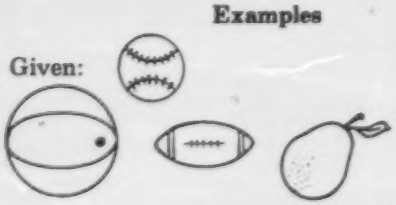
	Preschool and K	1	2	3	4	5	6	7	8
ITEMS	1 to 4	1 to 8	5 to 13	7 to 19	13 to 29	19 to 43	34 to 47	37 to 55	37 to 57

Objectives

1. Match models with other objects having the same shape.

Examples

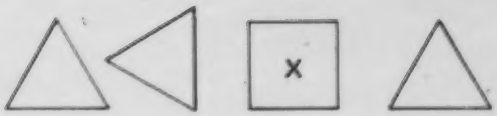
1. Given:



Show those having the same shape.

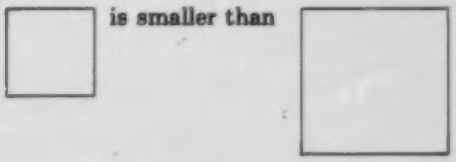
2. Discriminate among shapes and select the one that is different.

2.



3. Compare the size of geometric figures with the same shape.

3.

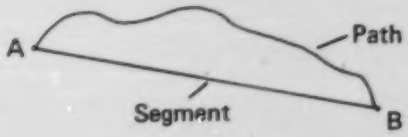


4. Identify geometric figures such as: triangle, rectangle, square and circle.

4. Find objects in the room with the given shape after having worked with models of the named figures.

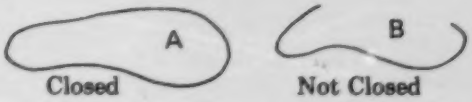
5. Demonstrate and discuss paths and shortest paths (segments).

5.



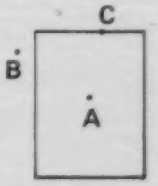
6. Identify closed and open (not closed) figures.

6.



7. Identify points that are inside, outside and on a figure.

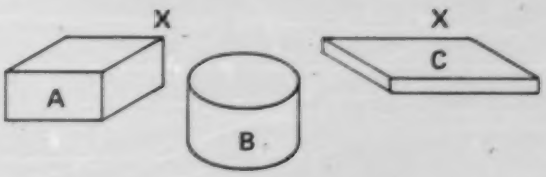
7.



A is inside the figure.
B is outside the figure.
C is on the figure.

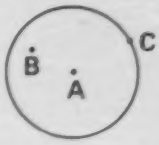
8. Match models of common shapes such as cones, spheres, cylinders and rectangular solids with objects having the same shape.

8.



9. Identify the center of a circle.

9.



Point A is the center of the circle.

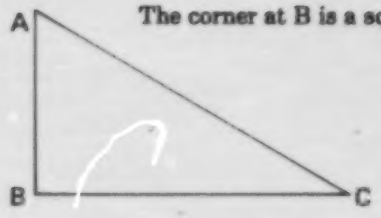
10. Identify corners and sides of figures.

10.

This figure has three sides.

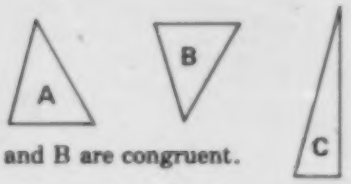
This figure has five corners.

11. Identify square corners (right angles) in a figure.

11. 

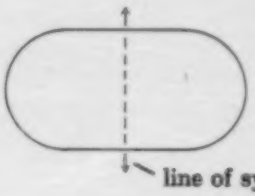
The corner at B is a square corner.

12. Identify congruent figures (same size and shape).

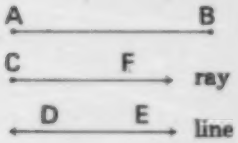
12. 

A and B are congruent.

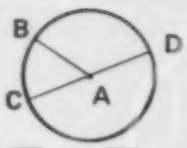
13. Demonstrate lines of symmetry.

13. 
line of symmetry

14. Name and demonstrate segments, lines and rays.

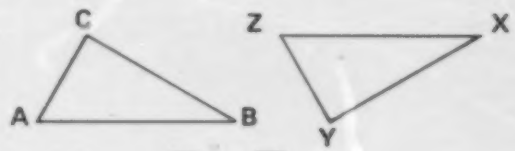
14. 

15. Identify and demonstrate radius and diameter of a circle.

15. 

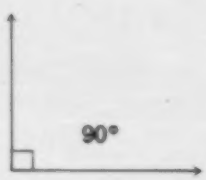
$\overline{AB}$, $\overline{AC}$ and $\overline{AD}$ are radii
$\overline{CD}$ is a diameter.

16. Identify and demonstrate corresponding parts of congruent figures.

16. 

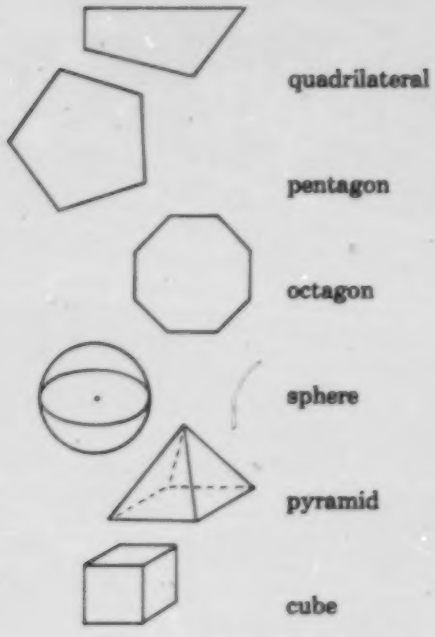
A ↔ Z $\overline{AB}$ ↔ $\overline{ZX}$
B ↔ X $\overline{BC}$ ↔ $\overline{XY}$
C ↔ Y $\overline{CA}$ ↔ $\overline{YZ}$

17. Draw a right angle and check by comparison with a known right angle.

17. 

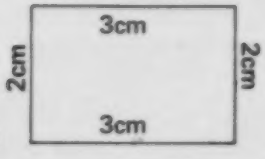
18. Identify geometric shapes such as quadrilaterals, pentagons, octagons, cubes, pyramids and spheres.

18.



19. Find the perimeter (distance around) some geometric figures.

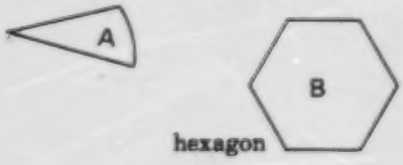
19.



perimeter = 10cm

20. Identify polygons (closed figures formed by line segments).

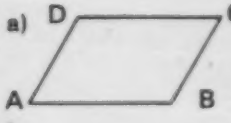
20. Figure A is not a polygon. Figure B is a polygon.



hexagon

21. Identify models of planes.

21. a)

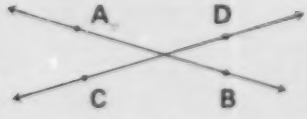


plane ABCD

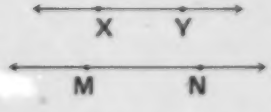
b) The ceiling or wall of the classroom.

22. Discriminate between intersecting and parallel lines.

22.



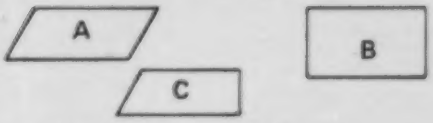
$\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect



$\overleftrightarrow{XY}$ and $\overleftrightarrow{MN}$ are parallel

23. Identify parallelograms.

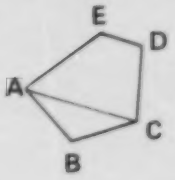
23.



A and B are parallelograms.

24. Identify diagonals of polygons.

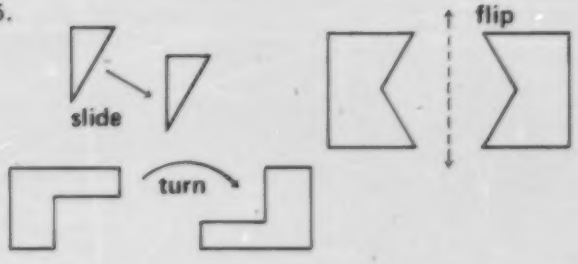
24.



$\overline{AC}$ is a diagonal of figure ABCDE.

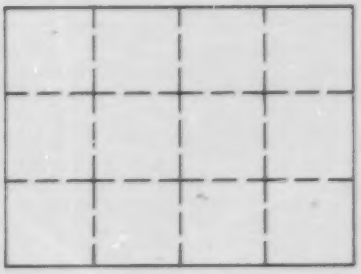
25. Identify and use transformations (slides, flips and turns).

25.

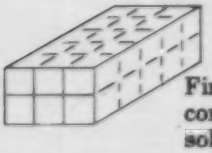


26. Find areas and volumes by counting.

26.



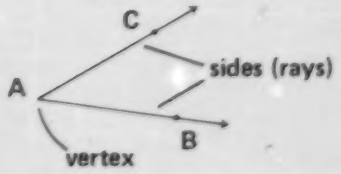
12 square units



Find the number of cubic units contained in the rectangular solid.

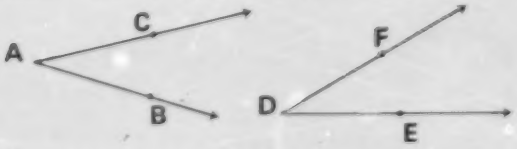
27. Identify angles and name their parts.

27.



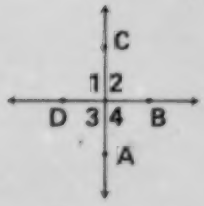
28. Determine congruent angles by fitting.

28. Show $\angle BAC$ congruent to $\angle EDF$ by fitting.



29. Indicate that two intersecting lines are perpendicular by showing all the angles formed are congruent.

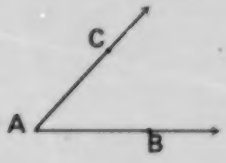
29.



Show $\overleftrightarrow{AC}$ perpendicular to $\overleftrightarrow{DB}$ by showing angles 1, 2, 3 and 4 congruent.

30. Measure angles using a protractor.

30.



measure of $\angle CAB = 47°$.

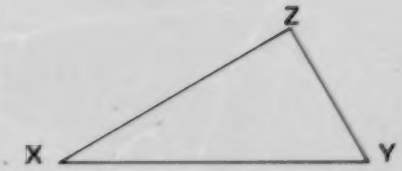
31. Identify right, acute, and obtuse angles.

31.

Right Acute Obtuse

32. Measure the angles of a triangle to show that the sum is 180°.

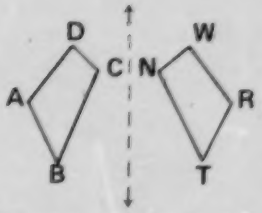
32.



$m\angle X + m\angle Y + m\angle Z = 180°$

33. Recognize that rigid motion of a shape (slides, turns, flips) results in a congruent shape and use this fact to verify the congruence of shapes.

33.



Show by a flip that figure ABCD is congruent to figure RTNW.

34. Identify the corresponding parts of congruent figures and show that they are congruent.

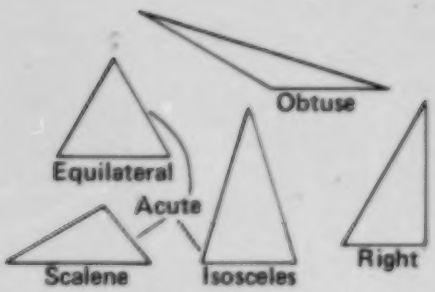
34. From the figures for example 33 show that the corresponding parts are:

$\overline{AB} \leftrightarrow \overline{RT}$	$\angle A \leftrightarrow \angle R$
$\overline{BC} \leftrightarrow \overline{TN}$	$\angle B \leftrightarrow \angle T$
$\overline{CD} \leftrightarrow \overline{NW}$	$\angle C \leftrightarrow \angle N$
$\overline{DA} \leftrightarrow \overline{WR}$	$\angle D \leftrightarrow \angle W$

By measuring or matching show that these parts are congruent.

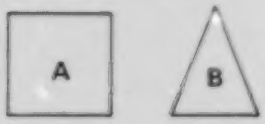
35. Recognize and name scalene, isosceles, and equilateral triangles; right, acute, obtuse triangles.

35.



36. Determine whether a polygon is regular or not.

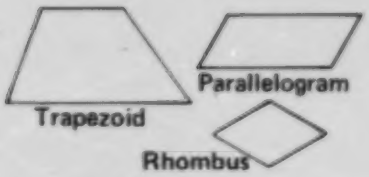
36.



A is a regular polygon
B is not

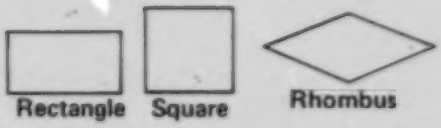
37. Recognize and name different quadrilaterals.

37.



38. Recognize and name special parallelograms. Describe the similarities and differences.

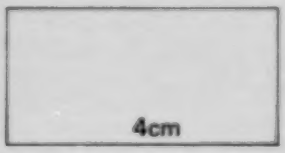
38.



rhombus: a parallelogram with all sides equal.
square: a rectangle with all sides equal.
rectangle: a parallelogram with all angles equal to 90 degrees.

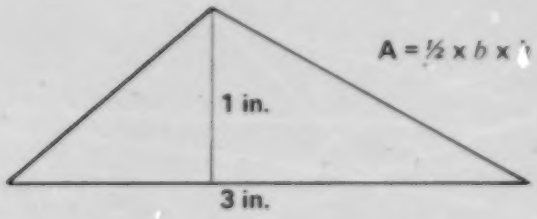
39. Find area by computation through the use of formulas.

39.



$A = l \times w$

area = 4 cm x 2 cm = $8cm^2$

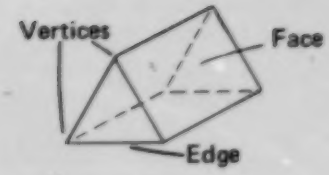


$A = \frac{1}{2} \times b \times h$

area = 1/2 x 1 in. x 3 in. = 1 1/2 sq. in.

40. Recognize and count the faces, edges and vertices of geometric figures.

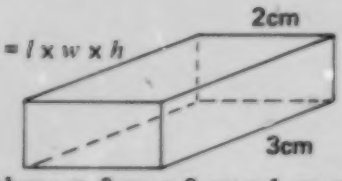
40.



The figure has 5 faces, 6 vertices and 9 edges.

41. Find volumes of selected geometric figures by computation using formulas.

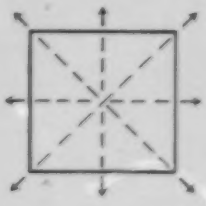
41. $V = l \times w \times h$



Volume = 3 cm x 2 cm x 1 cm = $6cm^3$

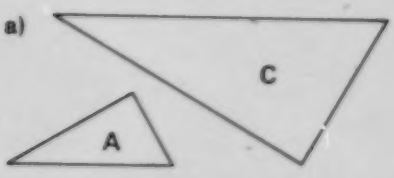
42. Analyze the flip and turn symmetries of shapes. Look for symmetries in real situations.

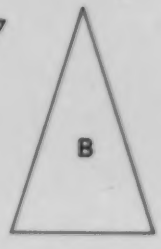
42. Indicate the lines and points of symmetry in this square and in this tree.



43. Classify shapes according to similarity and draw shapes similar to a given one.

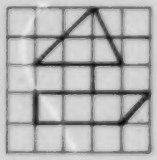
43. a)



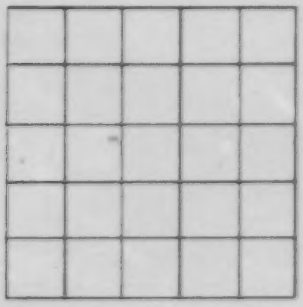


Figures A and C are similar

b)



Construct a similar figure to the one above on this grid.



44. Determine geometric figures that are polyhedrons by checking for flat or curved surfaces.

44.

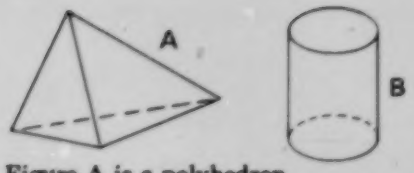
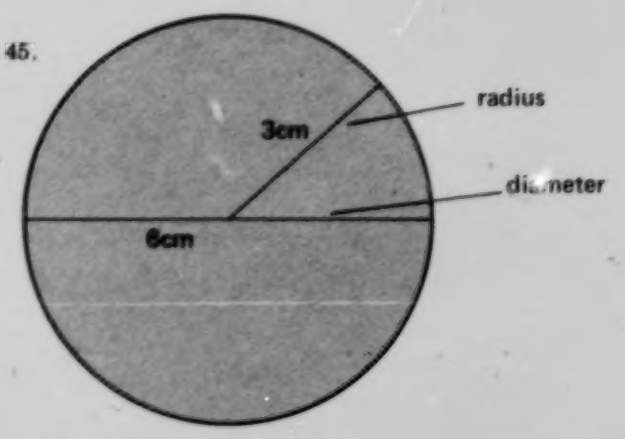


Figure A is a polyhedron
Figure B is not.

45. Find the area and circumference of a circle by the use of a formula; (π defined)

45.



Circumference is the distance around the circle.

Area is the measure of the shaded region enclosed by the circle.

Pi is the ratio of the circumference of a circle to its diameter. ($\pi = c/d$)

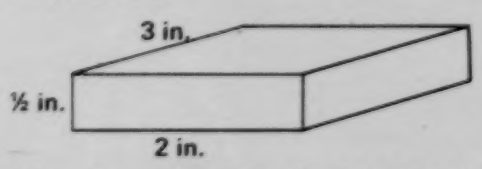
Determine approximate values of pi by experimentation.

$C = \pi \times d$
$C \approx 3.14 \times 6$ cm
$C \approx 18.84$cm

$A = \pi \times r^2$
$A \approx 3\ 1/7 \times 3$cm $\times 3$ cm
$A \approx 28\ 2/7$cm^2

46. Compute the surface area of some selected geometric figures.

46.



Surface Area = $2 \times (1/2 \times 2) + 2 \times (1/2 \times 3) + 2 \times (2 \times 3)$
$= 2 + 3 + 12$
$= 17$ sq. in.

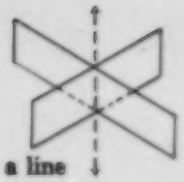
47. Complete selected geometric constructions using a compass and a straightedge.

47. a) 1. Copy an angle and a line segment.
2. Bisect an angle and a line segment.
3. Perpendicular to a line from a point on the line and from a point not on the line.
4. A line parallel to a given line through a point not on the line.

b) Construct triangles given three necessary and sufficient parts.

48. Identify the intersection of geometric figures.

48. a) Two Planes



intersection: a line

intersection: empty set

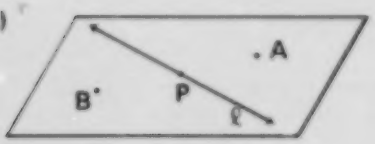
intersection: a plane

b) Situations similar to the above examples exist for the intersections of:

1) two lines
2) a line and a plane

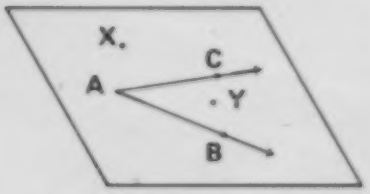
49. Identify and use terminology associated with line and plane separation.

49. a)



Line l divides the plane into two half-planes; point P divides line l into two half-lines.

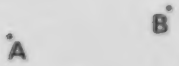
b)



Point X is in the exterior of $\angle BAC$; Point Y is in the interior of $\angle BAC$.

50. Demonstrate conditions necessary to determine a line and a plane.

50. a)



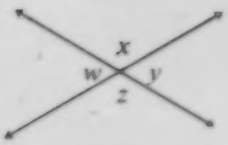
Distinct points A and B determine one and only one line.

b)

Three non-collinear points determine one and only one plane.

51. Identify angles formed by two intersecting lines and the relation between the angles.

51.

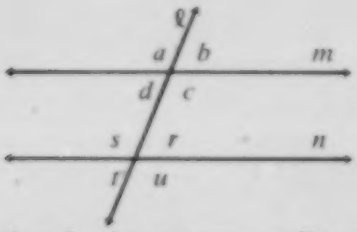


∠x and ∠z are vertical angles and have equal measures.

∠y and ∠z are supplementary angles and the sum of their measures is 180°.

52. Identify pairs of angles formed when a transversal intersects a pair of lines; determine their relation if the lines are parallel.

52.

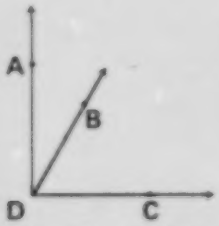


a) ∠b and ∠r are corresponding angles.
∠c and ∠s are alternate interior angles.

b) The measures of the angles are equal if the lines are parallel.

53. Define and use complementary angles.

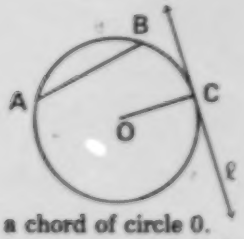
53.



∠ADB and ∠BDC are complementary and the sum of their measures is 90°.

54. Identify and use other items associated with a circle.

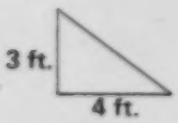
54.

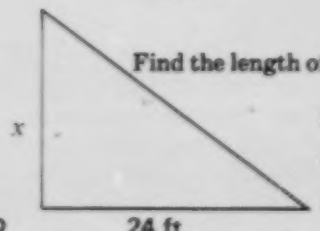


$\overline{AB}$ is a chord of circle 0.
Line *l* is tangent to circle 0 at point C.
$\overset{\frown}{AB}$ is an arc of circle 0.

55. Recognize the necessary and sufficient conditions for similarity of geometric figures, and use these properties to solve problems.

55.





Find the length of the segment labeled x.

3/4 = x/24

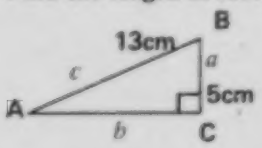
4x = 72

x = 18 ft.

56. Recognize and use the Pythagorean Theorem with selected right triangles. (Pythagorean triples)

56. a) Find the length of the hypotenuse if the two legs have measures of 6 in. and 8 in.

b) Find the length of $\overline{AC}$.



$c^2 = a^2 + b^2$

$c^2 - a^2 = b^2$

$(13)^2 - (5)^2 = b^2$

$169 - 25 = b^2$

$144 = b^2$

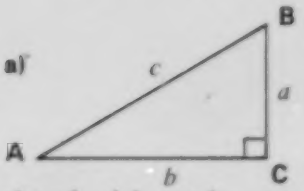
$\sqrt{144} = \sqrt{b^2}$

$12\text{cm} = b$

57. Recognize and use selected trigometric functions for a right triangle.*

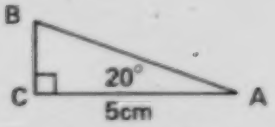
*Optional

57. a)



sine of $\angle A = a/c$
cosine of $\angle A = b/c$
tangent of $\angle A = a/b$

b)



Find the length of $\overline{BC}$.

$\tan 20° = \overline{BC}/5\text{cm}$
$.364 = \overline{BC}/5\text{cm}$
$(.364)\ 5\text{cm} = \overline{BC}$
$1.820\text{cm} = \overline{BC}$

Measurement

The strand on measurement includes both premeasurement and measurement concepts and skills. Its scope has been expanded to include the metric system as an integral part of the strand. Refer to page 76 for a comprehensive statement concerning the teaching of the metric system.

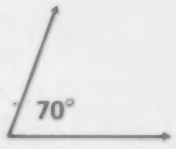
The classroom teacher should understand that the metric system, to be learned effectively, must be learned by the student as a separate system. It is appropriate to help the student learn comparisons between the metric and customary systems such as "one litre is a little more than one quart" and "2.5 centimetres is a little less than one inch." However, it is not appropriate for the student, especially those in the primary and intermediate grades, to perform conversions from one system to another.

The concepts of perimeter, area and volume from metric geometry are included in the strand titled "Geometry."

SUGGESTED GRADE LEVELS

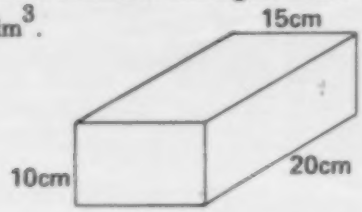
	Preschool and K	1	2	3	4	5	6	7	8
	1 to 4	1 to 8	6 to 13	10 to 19	16 to 20	17 to 23	21 to 26	25 to 29	25 to 31

Objectives	Examples
1. Compare two objects as to specified characteristics. (length, weight [*mass*], volume, temperature).	1. a) Which stick is longer? b) Which is heavier, the ball or the balloon? c) Which glass of milk has more in it? (same size glass) d) Which is colder, the piece of ice or the pencil?
2. Use informal units of measurement to describe characteristics of a given object. (length, weight, volume, temperature).	2. a) How many pencil lengths wide is the desk? b) How many *glasses* of Kool-aid are in this pitcher? c) Tell me how many nickels this chalkboard eraser weighs.
3. Identify order of three objects as to specified characteristics. (length, weight, volume, temperature).	3. a) Which of the three boys is tallest? b) Tell me which book is heaviest. c) Which container has the most water? d) In what season are the days the hottest?
4. Recognize place relationships, speed comparisons.	4. a) Which tree is farther from us? (two trees) b) Which car is traveling faster? (two cars)
5. Identify coins and value. (penny, nickel, dime).	5. a) Show me the penny. b) Show me the nickel. c) Show me the dime. d) Which of the three coins is worth the most?
6. Recognize the need for standard units of measure by making simple measurements using non-standard units.	6. a) How many steps long is the classroom? b) Use your hand span to measure the width of the chalkboard.
7. Make simple linear measurements to the nearest unit in centimetres, decimetres, metres, inches, feet and yards.	7. a) How long is this pencil to the nearest centimetre? b) How long is the playground to the nearest yard?
8. Tell time to the nearest hour and half hour and name seasons, months and days of the week.	8. a) Large calendar with removable month names is brought up to date each day, and future plans are made with its help. b) It is about time for lunch. Where is the small hand on the clock when we come to school? When we leave to go home? Egg or oven timer can be used to mark time intervals.
9. Tell time by season, month, and hour.	9. a) Look at the calendar and tell me what day it is. b) What season immediately follows spring?
10. Tell temperature in degrees, Celsius and Fahrenheit.	10. To the nearest degree, what is the temperature on the thermometer?
11. Measure the weight (mass) of an object to the nearest kilogram and pound.	11. Look at the scale and tell me how many kilograms or pounds Billy weighs.

12. Measure capacity in litres, cups, pints, quarts and gallons.	12. To the nearest litre, how many litres of water are in this bucket?
13. Compare the following units: a) cm, dm, m b) in., yd., ft.	13. a) There are _____ centimetres in a decimetre. b) There are _____ decimetres in a metre. c) There are _____ inches in a foot. d) There are _____ feet in yard.
14. Recognize that the time represented between succeeding numerals on a clock represents a five minute time interval.	14. How many minutes have gone by when the minute hand has moved from 12 to 4 on the clock.
15. Measure length to the nearest half-inch.	15. Use rulers marked to the nearest quarter-inch to measure objects. Measure any line segment to the nearest half-inch.
16. State the value in cents of a penny, nickel, dime, quarter, half dollar, dollar.	16. How many one cent bubble gums can I buy for a nickel? For a dime?
17. Tell time to the nearest minute.	17. Look at the clock and tell me what time it is.
18. Convert given measures to equivalent measures within the same measurement system.	18. 5 km = 5000m 10,560 ft. = 2 mi.
19. Choose an appropriate measurement unit for a given situation.	19. What unit would you use to measure the distance around your head?
20. Measure length to the nearest quarter-inch or centimetre.	20. Measure any line segment to the nearest quarter-inch or centimetre.
21. Add and subtract measures without regrouping.	21. If one piece of rope is 3 feet 4 inches long, and another piece is 4 feet 2 inches long, what is their combined length?
22. Measure lengths to the nearest sixteenth-inch and to the nearest millimetre.	22. Draw a line segment on your paper. Now measure it to the nearest 16th-inch and to the nearest millimetre.
23. Add, subtract and multiply measures with regrouping.	23. If I have a piece of rope 4 feet 9 inches long, and a second piece of rope 3 feet 5 inches long, how long are the two pieces together? 4′ 9″ 3′ 5″ 8′ 2″
24. Using a protractor, measure an angle or construct an angle.	24. a)  b) Draw an angle of 40°
25. Make a scale drawing of a given figure or object.	25. Using a centimetre grid draw the floor plan of the classroom using a scale of 1 cm = 50 cm.

26. Calculate the area and volume of common figures and convert to equivalent measures within the same system.

26. Find the volume of this figure in cm^3, then convert to dm^3.



Given that 1 cm^3 has a capacity of 1 ml find the capacity of the figure.

27. Conduct experiments involving the use of various standard units.

27. a) Find the perimeter of the playground or football field.
b) Find the volume of a fish tank.

28. Recognize that error is inherent in measurement, and estimate variations caused by error.

28. In measuring a segment to the nearest inch, the greatest possible error is _____ inch.

29. Add, subtract, multiply and divide measures with regrouping.

29. If you divide 2 feet 3 inches of ribbon equally among 3 girls, how much ribbon will each girl receive?

30. Recognize the existence of error in measurement, and apply the concepts of absolute and relative error.

30. Given two measures, 304 feet and 151 centimetres.
a) Wnat is the greatest possible error in each measure?
b) What is the relative error in each measure?

31. Make calculations involving measurements using rules for significant digits.

31. a) 6.86 cm
7.3 cm

14.16 which would be reported as 14.2 cm.

b) 2.4 cm
x1.7 cm

4.08 which would be reported at 4.1 cm^2.

Problem-Solving and Number Sentences

Problem-solving means putting mathematics to use. The ability to solve problems is basic not only to mathematics but to a wide variety of daily experiences. The use of mathematics should not be regarded as an afterthought to the development of concepts and skills. Rather, applications of mathematics must be a fundamental part of many lessons.

There are many types of problems which should be explored in the mathematics classroom. Among them is the type most commonly found in exercise sets of textbooks. The purpose of this type of problem-solving is to provide drill and practice of computational skills. One example is the following open sentence: $7 - 3 = \square$.

An extension of this type of problem-solving involves the translation of a story or word problem into a mathematical sentence. As each operation and additional kinds of numbers are introduced, students need practice with a) drawing pictures or diagrams to illustrate and describe the given situation, b) writing the mathematical sentence which symbolizes the situation, c) solving the mathematical problem and d) interpreting the mathematical solution in terms of the given problem.

Another variety of problem-solving may not require the use of any formula or mathematical sentence nor be directly connected with any specific mathematical content. Often classified as "brain teasers", these problems are usually highly motivating and unfortunately reserved for only the gifted. These problems are designed to encourage creative and logical thinking and should be a part of every child's learning experiences. To assist students in this type of problem-solving process, the following steps are recommended: a) Try to understand the problem by sketching some figures, diagrams, or tables summarizing the information given; b) Determine a planning strategy on how to get started by breaking the problem into small increments and estimate the solution; c) Carry out the plan of attack, reflecting occasionally on whether the approach is contributing to the solution; d) Analyze whether the approach you used seems to be the most logical.

The last category of problem-solving involves a more challenging, real world situation that is geared to the student's thinking level and therefore more meaningful. This problem-solving process involves interdisciplinary investigations of problems that arise in everyday living. The objective is to help students learn how to collect and interpret data from actual experiences.

Because there is no single method for solving a particular problem nor a single method which is best for everyone, students need many experiences with deciding what a problem involves. Problem situations should involve a wide variety of settings with any of these relating to the students' own experiences and interests. It is not enough that students know how to add, they must also know when and what to add.

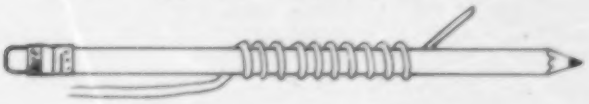
Problem-Solving and Number Sentences: Performance Objectives

SUGGESTED GRADE LEVELS

	K	1	2	3	4	5	6	7	8
ITEMS	1 to 2	1 to 9	3 to 15	10 to 19	14 to 23	16 to 28	24 to 34	24 to 37	24 to 44

Objectives	Examples
1. Act out a story involving addition as the teacher tells the story.	1. Tom puts three chairs at the table. Mary puts one chair at the table. How many chairs are now at the table?
2. Draw a picture that illustrates a story told by the teacher.	2. John has four marbles and Billy has three marbles. Who has more marbles? Draw a picture showing the story.
3. Tell how to solve a word problem.	3. Mary has three dolls and Jane has three dolls. How can we find out how many dolls there are in all?
4. Identify an open sentence which represents a given story problem. Use addition or subtraction with sums no larger than 10.	4. There are five plates on the table. One plate is broken. Draw a ring around the sentence which shows the number of plates left. $5 + 1 = \square$ $5 - 1 = \square$
5. Write a number sentence for a given story problem.	5. Fred has two candy bars and Sue has five. How many candy bars are there in all? Student should be able to write $2 + 5 = \square$
6. Illustrate an addition or subtraction number sentence by telling a story and by making a drawing.	6. Tell a story which can be shown by the sentence $3 + 4 = 7$.
7. Determine what information could be found presented with an open ended situation.	7. Jim has six marbles and Henry has four. What can we find out?
8. Solve open sentences using the addition facts through sums of 10.	8. Write the number in the box which will make the sentence true. $2 + 3 = \square$ $\quad 8 - 3 = \square$ $2 + \square = 6$ $\quad 7 - \square = 4$
9. Through experimentation and observation, calculate the result of a given problem.	9. Wrap a piece of string around your pencil 10 times. Estimate how many centimetres of string

could be wrapped around your pencil 50 times.



10. Translate written relations to mathematical sentences and determine the solution.

10. Chris found that five plops and five blops have a mass of 100 grams. Terrie found that six plops and two blops have a mass of 100 grams. What is the mass of 10 plops and 10 blops? How can Chris and Terrie find the mass of one plop?

11. Write in the symbols $>$, $<$, or $=$ to make true sentences.

11. Use the symbols $>$, $<$, or $=$ to make the following true sentences.

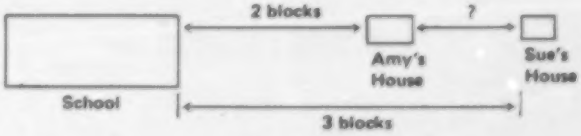
a) $3 \square 5$ b) $3 - 2 \square 1$
c) $3 + 2 \square 1 + 3$

12. Solve open sentences with more than one place holder.

12. Find some numbers which will make the following $\square + \square = 8$ a true sentence.

13. Illustrate a word problem by a drawing, translate into an open sentence and solve. These could involve money problems.

13. Amy and Sue walk home from school together. Amy's house is two blocks from school and Sue's house is three blocks from school. How far does Sue walk by herself?



$2 + \square = 3$ or $3 - 2 = \square$

14. Solve open sentences using addition facts with sums through 18.

14. Write the number in the box which will make the sentences true.

$4 + 9 = \square$ $\square + 5 = 17$ $13 - \square = 7$

15. Write addition sentences which result from given subtraction statements and vice versa.

15. Given $3 + 8 = \square$, then $\square - 3 = 8$ and $\square - 8 = 3$.
Given $\square - 3 = 5$, then $3 + 5 = \square$ and $5 + 3 = \square$.

16. Determine whether a mathematical sentence is true or false.

16. Determine whether the following are true or false.

a) $32 + 48 = 175 - 95$
b) $87 + 46 = 77 + 77$

17. Determine a procedure for product testing.

17. In groups, design and evaluate which peanut butter is the best buy from a set of four or five different brands.

18. Solve a word problem without the use of pencil and paper.

18. One turn of a bicycle wheel moves the bicycle ahead about two metres. Estimate how many turns of the tire are guaranteed by the manufacturer.

19. Write multiplication number sentences which describe a given array.

19. Given
x x x x
x x x x
x x x x
then $3 \cdot 4 = 12$

20. List a rate equivalent to a given rate.

20. Bill will trade two marbles for one baseball card. Joe has 12 baseball cards. How many marbles will Bill need to get Joe's baseball cards?

21. Write number sentences which compare numbers.

21. Sam earns 60 cents a week and Julie earns 40 cents. Write a number sentence comparing their earnings.

22. a) Write equations given addend, addend and sum.
b) Write equations given two factors and a product.

22. a) Given that three is an addend, seven is an addend and represents the sum, write an equation showing this relationship.
b) Write an equation which shows that three is a factor, □ is a factor and 15 is the product.

23. Make a study of student food waste in the school cafeteria.

23. In groups, survey and analyze food wasted in the school cafeteria. Make suggestions on how food waste can be minimized.

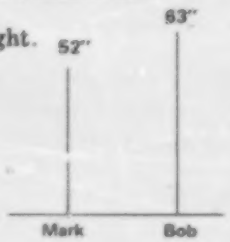
24. Tell how to solve a word problem when the operation of addition, subtraction, or multiplication is used.

24. Andy played 16 records for a party. Each one lasted 18 minutes. How many minutes of music were on the records?

25. Draw a diagram, write an open sentence and solve a one step story problem.

25. How many inches taller is Bob than Mark if Mark is 52 inches tall and Bob is 63 inches tall?

h = difference between height.
$52 + h = 63$
or
$63 - 52 = h$



26. Tell a story for a given number sentence.

26. Tell a story to illustrate:
a) $27 \cdot 32 = 864$
b) $3 \cdot 52 = 156$

27. Write a division sentence with results from a given multiplication statement.

27. Given $7 \times 4 = □$, then $□ \div 7 = 4$ and $□ \div 4 = 7$.
Given $20 \div 4 = □$, then $4 \times □ = 20$.

28. Write sentences which compare fractional numbers.

28. Write sentences for:
a) 1/2 is greater than 1/3.
b) 6/9 and 2/3 represent the same number.

29. Write open sentences using letters as placeholders.

29. The area of a rectangle is 84 square centimetres and one side of the rectangle is 7 centimetres, find the length of the other side.
Students should write $7 \cdot t = 84$ where t is the length of a side of the rectangle.

30. Tell how to solve a problem without numbers.

30. ? seats in an auditorium
? seats in each row
? number of rows

31. Solve open sentences using placeholders in every possible position and using the inverse operations.

31. What numbers make the following true?
$□ + 36 = 84$ $3 \cdot n = 36$
$□ = 84 - 36$ $n = 36 \div 3$

32. Write sentences which compare decimal numerals.	32. Write a sentence showing the relationship between 0.3 and 0.32.
33. Determine the truth or falsity of sentences using $<$, $>$, $=$, $\neq$, $\not<$, $\not>$, whole numbers, fractions, decimals, and the four fundamental operations.	33. State whether each of the following is true or false and write a true sentence if the given one is false. $43 \times 6 \neq 240$ $23 \cdot 35 > 735$
34. Apply the concept of proportion in solving problems.	34. a) If a helicopter travels 240 miles in three hours, how long does it take to travel 140 miles? b) Two numbers x and y vary directly. If x is four when y is six, find x when $y = 18$.
35. Find the third quantity, given two of the three following quantities: base, rate, percentage.	35. a) 10 percent of 60 is what number? b) What percent of 60 is 24? c) 15 is 10 percent of what number?
36. Determine the solution set for selected open sentences. Solution sets may be sets of one member, infinite sets, or the empty set.	36. Solution set for $543 - n = 543$ is 0. Solution set for $n = n + 1$ is the empty set. Solution set for $p + 8 = 5$ is empty if only whole numbers are used. Solution set for $n + 3 = 3 + n$ consists of all the numbers in the replacement set.
37. Solve linear equations using two or more steps. Solutions can be whole numbers, fractions or decimals.	37. Solve and check. a) $2n + 5 = 19$ b) $2(n + 5) = 17$
38. Estimate an answer to an open sentence or word problem.	38. There are 1,008 seats in an auditorium and the seats are arranged in rows of 21 seats each. Estimate the number of rows. Students might estimate 50 rows since 1,000 divided by 20 is 50.
39. Draw a picture, write an open sentence, and solve a word problem involving two or more steps.	39. A rectangular building is surrounded by a walk of uniform width. If the building is 20 feet by 30 feet and the walk is 2 feet wide, find the dimensions of the building and walk.
40. Solve open sentences involving integers.	40. Solve. a) $p + 8 = 3$ b) $-7n = 42$
41. Solve inequalities.	41. a) What is the largest whole number r such that $r < 7/4$? b) Solve $2n + 3 > 7$.
42. Solve open sentences involving rational numbers.	42. Solve. a) $2n + 1 = -8$ b) $3 - 2n > -8$
43. Solve equalities and inequalities involving real numbers.	43. Solve. a) $-\sqrt{2} + x = 3$ b) $5n < -\sqrt{2}$
44. Solve formulas for other variables.	44. Express w in terms of A and l if $A = lw$.

Graphing and Relations

A graph is a way of depicting information. There are many opportunities in the elementary school for compiling data and showing this information on bar, line, picture and circle graphs. In addition, many examples of these graphs can be found in newspapers and periodicals. Since these graphs are used in interpreting statistical data, objectives involving them also are found in the strand titled, "Probability and Statistics."

Another aspect of graphing is the location of a point on a grid and the consideration of relations and functions. Since relations can be described by a) a table, b) a list of ordered pairs, c) points on a rectangular coordinate grid and d) a mapping diagram, students need to be aware of all these methods and be able to derive one from another.

Graphing and Relations: Performance Objectives

SUGGESTED GRADE LEVELS

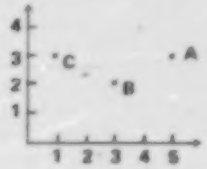
	K	1	2	3	4	5	6	7	8
ITEMS	1 to 2	1 to 4	1 to 6	3 to 10	7 to 15	10 to 19	15 to 22	10 to 14 and 19 to 30	10 to 14 and 19 to 33

Objectives	Examples
1. Gather, organize, graph and interpret some data.	1. Have children use a length of yarn equal to the length around their waist. Make a graph by suspending each of these from a piece of masking tape on the wall. Questions such as who has the smallest waist, who has the same size waist, etc., can be discussed.
2. Locate points on a number line which correspond to whole numbers one through 10.	2. Make a number line on the floor with masking tape. Children can show a point on the number line which corresponds to the number. (Can relate to the thermometer).
3. Locate points on a number line that represent whole numbers which are solutions to equalities and inequalities.	3. Show on the number line the whole number which satisfies. a) $3 + 4 = \square$ b) $\Delta < 4$.
4. Make a table of values which satisfies an open sentence in two variables.	4. Make a table of whole number values to make $\square + \Delta = 6$ a true statement.
5. Identify the ordered pair of numbers which locate a particular object within the classroom.	5. Use the class seating chart to: a) Give the row (horizontal) and seat (vertical) location of each student's location. b) Identify the student who sits in row three, seat two.
6. Give directions for getting from one place to another on a map which makes use of *only* horizontal and vertical paths.	6. a) Make a street map using the streets in the vicinity of the school. Have the students give as many ways as possible to get from one intersection to another by using the streets.

b) Discuss the shortest route and express as an *ordered pair* of whole numbers emphasizing the horizontal first.

7. Locate ordered pairs of whole numbers on a grid. Name and write the ordered pair which corresponds to a given point on the grid.

7. Given the grid below:
   a) Locate the point (3,1)
   b) Give the ordered pair which names point A.



8. Given a rule for a function and one of the elements, find the other.

8. a) Find the outputs for the function when the rule is "add 3" and the inputs are 2, 4, 5 and 6.
   b) The outputs for a function are 3, 5, 6 and 9 when the rule is subtract 4. What are the inputs?

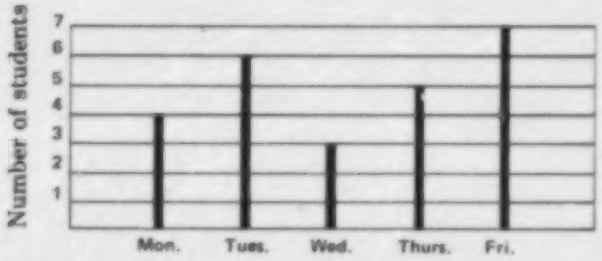
9. Given a set of specific pairs of elements for some function, find the rule for that function.

9. What's the rule for the function which has inputs and outputs as given in the table below?

input	output
4	2
5	3
6	4
9	?

10. Interpret the information in bar and picture graphs.

10. Students bringing lunch each day



11. Identify the kinds of graphs used in the real world.

11. Make a collection of graphs from newspapers and magazines. Identify the kind of graph and interpret the information.

12. Construct bar and picture graphs.

12. Collect weather information for a period of time. Show this information on both bar and picture graphs.

13. Interpret and construct line graphs.

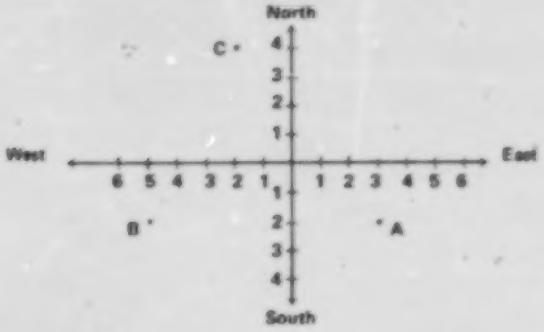
13. Collect weather information for a period of time. Show this information on a line graph.

14. Compare bar, line and picture graphs of the same information giving advantages and disadvantages.

14. Use the graphs prepared for items 12 and 13 above to discuss which graph gives the clearest and poorest picture of the information.

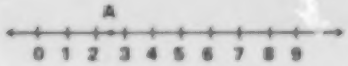
15. Locate ordered pairs of whole numbers on a four quadrant grid. (Latitude and longitude may be used but remember this gives the vertical and then the horizontal. You may wish to give it as longitude, latitude)

15. Given the grid below:
a) Locate the point (left 3, down 2)
b) Name the location of point C.



16. Locate the points on a number line which correspond to positive rational (fractional) numbers.

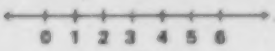
16. a) Locate the point on the number line below which represents each of the numbers 3/4, 3/3 and 5/2



b) Point A on the number line represents what number?

17. Indicate the part of the number line which represents the solution set for equalities and inequalities.

17. a) Shade the part of the number line below which represents numbers satisfying $x > 3$.

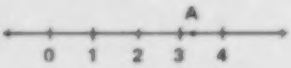


b) Does 3 satisfy $x > 3$? How is this represented on the graph?

c) Give some numbers not identified on the number line which satisfy $x > 3$.

18. Locate and name points on a number line using decimal notation.

18. a) Locate a point of the number line below which represents 1.50.



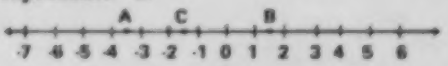
b) Point A on the number line represents what number? (Use decimal form).

19. Determine:
a) The rule for a function given a set of specific pairs of elements.
b) The missing element, given the rule for a function and one of the elements.
(All four operations should be utilized).

19. Fill in the missing parts for the function machine below.

Rule
?

input	output
2	5
5	11
8	17
1/2	?
?	31
2.5	?

20. Choose an appropriate scale for constructing a graph using given information. Graph the information using two or more different scales and compare the resulting graphs.	20. Miss Jones' class sold light bulbs. David sold 12, Lisa sold 24, John sold 12, Nancy sold 30 and Jim sold 42. What scale could be used in graphing this information? Discuss the similarities and differences of the resulting graphs.
21. Construct a mapping diagram to illustrate a given relationship.	21. Given: "Jane ⟶ George" implies that George is Jane's brother. Construct a mapping diagram that shows brothers Harry and George are Jane's brothers.
22. Construct and interpret data on a circle graph. (Students will need to be able to use percentage and a protractor).	22. In a contest, Sally sold 30 boxes of candy, Jim sold 15, John sold 9 and Sue sold 6. Represent this information on a circle graph.
23. Locate points on a number line including negative rational numbers.	23. a) Locate the point on the number line below which represents -4.  b) Name the numbers represented by the points A, B and C.
24. Graph ordered pairs of rational numbers on a coordinatized plane.	24. Graph the following points using the same axes. (4,2.9); (-2.5,-1); (-3.24,0.5); (0.75,-2)
25. Graph the linear equation $ax + by + c = 0$ on a coordinate plane for a, b and c any rational number.	25. Graph: a) $x + y = 1$; b) $x = 4$; c) $y = 0$
26. Indicate the part of the plane which represents the solution set for equalities and inequalities and name some ordered pairs that belong to the solution set.	26. Graph $x + y > 1$. Name three ordered pairs that belong to the solution.
27. Identify and give examples of: axis, origin, quadrant, abscissa, ordinate, and coordinates.	27. Define abscissa. The point (-2.5,3) is in what quadrant?
28. Recognize the domain and range of a function defined by a set of ordered pairs.	28. Given a function defined by: $\{(1,6), (2,7), (4,10)\}$ List a) The domain b) The range
29. Use the function notation f(n).	29. For $f(p) = 1 - 3p$; evaluate $f(1/2)$.
30. Given the domain of a function and a rule for the function, list some ordered pairs for the function and determine whether a given ordered pair belongs to the function.	30. Given $f(x) = 2x + 3$ a) List three ordered pairs of the function. b) List the same pairs in table form. c) Does (6,11) belong to the function?
31. Distinguish between relations which are functions and those that are not.	31. Which of the following are functions and which are not functions. Each set of ordered pairs is complete, i.e., you need consider no other ordered pair. i) $\{(3,4); (3,2); (7,7)\}$. ii) $\{(4,9); (2,8); (6,8)\}$. iii) $\{(6,2); (5,2); (9.5)\}$.
32. Graph solutions of compound sentences on a coordinate plane.	32. Graph $y \geq x - 2$.
33. Graph irrational numbers on a number line.	33. Find the point on the number line which represents $\sqrt{2}$; which represents π .

Probability and Statistics

The statistics portion of this strand deals with collecting and interpreting data, thereby giving meaning to the data. The strand also deals with the probability associated with chance events. Predictions can be made from statistical sampling and probability analysis in which frequency charts and graphs may be used to provide a clear picture of the data. Therefore, objectives involving bar, line, picture and circle graphs are found in this strand as well as in the strand titled, "Graphing and Relations".

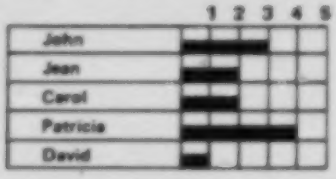
Probability and Statistics: Performance Objectives

SUGGESTED GRADE LEVELS

	K	1	2	3	4	5	6	7	8
ITEMS	1 to 4	1 to 5	3 to 6	5 to 9	7 to 11	9 to 15	12 to 20	16 to 24	19 to 26

Objectives	Examples
1. Collect, arrange and represent data.	1. Construct a block graph to show the birth months of classmates.
2. Record data using a frequency table and interpret the results.	2. Record the information from example one in a frequency table and answer the following: a) The month with most birthdays? b) The month with fewest? c) Were more children born in the fall or in the summer?
3. Make comparisons of likelihood.	3. Children discuss the likelihood of such things as: a) Drawing a red marble out of a bag containing a red marble and a green marble. b) A hexagonal spinner stopping on the red edge if each edge is marked with a different color. c) Winning a game with two competitors compared with winning a game with four competitors. d) Having snow on Christmas.
4. Compile and interpret information on a pictograph (picture graph).	4. Survey the number of pets in each family. Draw a *pictogram* or *pictograph* to illustrate the results. a) How many dogs were there? b) How many cats? c) Which pet was most common? d) Which pet was least common? e) Were there more mice than gerbils?
5. Find the average by leveling off.	5. Build a graph from a given set of data with blocks or tiles. Mark the outline of the block graph and move the blocks or tiles from the top until they have formed a rectangle which will give the "average line".
6. Gather, organize and interpret information.	6. a) Place an adding machine tape adjacent to a 150 centimetre tape and record the height of the children in the classroom. Then discuss the data using range, frequency, etc. without using the technical terms.

b) The number of hours each of five children in the classroom watched television last Saturday morning.



Who watched the most? The least? How much more did Patricia watch than David?

7. Make predictions from random sampling.

7. a) Figure out some way of telling how many books are in your classroom library without counting every one.
   b) Compute the approximate number of beans in a paper cup given another identical empty one and a weighing scale. Do not count every bean!

8. Use average as a number that helps describe a group.

8. Estimate the average weight (mass) of the children after observing each child being weighed.

9. Construct a frequency table from a collection of data and find the average and range.

9. Take the pulse rate of 20 students in your class. Record and group these rates. Find the average and range (least — most). Guess the teacher's pulse rate and then check it.

10. Predict simple outcomes.

10. If you pick a ball without looking from a bag of five green and one red, are you more likely to get a green ball or a red ball?

11. Construct and interpret bar, line and picture graphs.

11. Graph mathematics scores, other individual or class scores, and weather information for periods of days or weeks.

12. Count the number of possible arrangements of a set of different objects.

12. How many different arrangements in a row can David make of his three favorite football players' pictures? Of his four favorite football players?

13. Represent and count the number of elements in the sample space for a given event.

13. a) Toss a die; list the possible outcomes; tell how many outcomes exist.
   b) Toss a nickel and a dime; list the possible outcomes; tell how many outcomes exist.

14. Use fractions to represent probabilities.

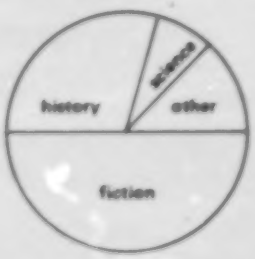
14. With six white balls and one red ball the chance of drawing a red ball on the first draw is 1-7 or 1/7.

15. State the probability that an event will occur.

15. A box contains seven red and five white balls. What is the probability of your drawing at random one white ball?

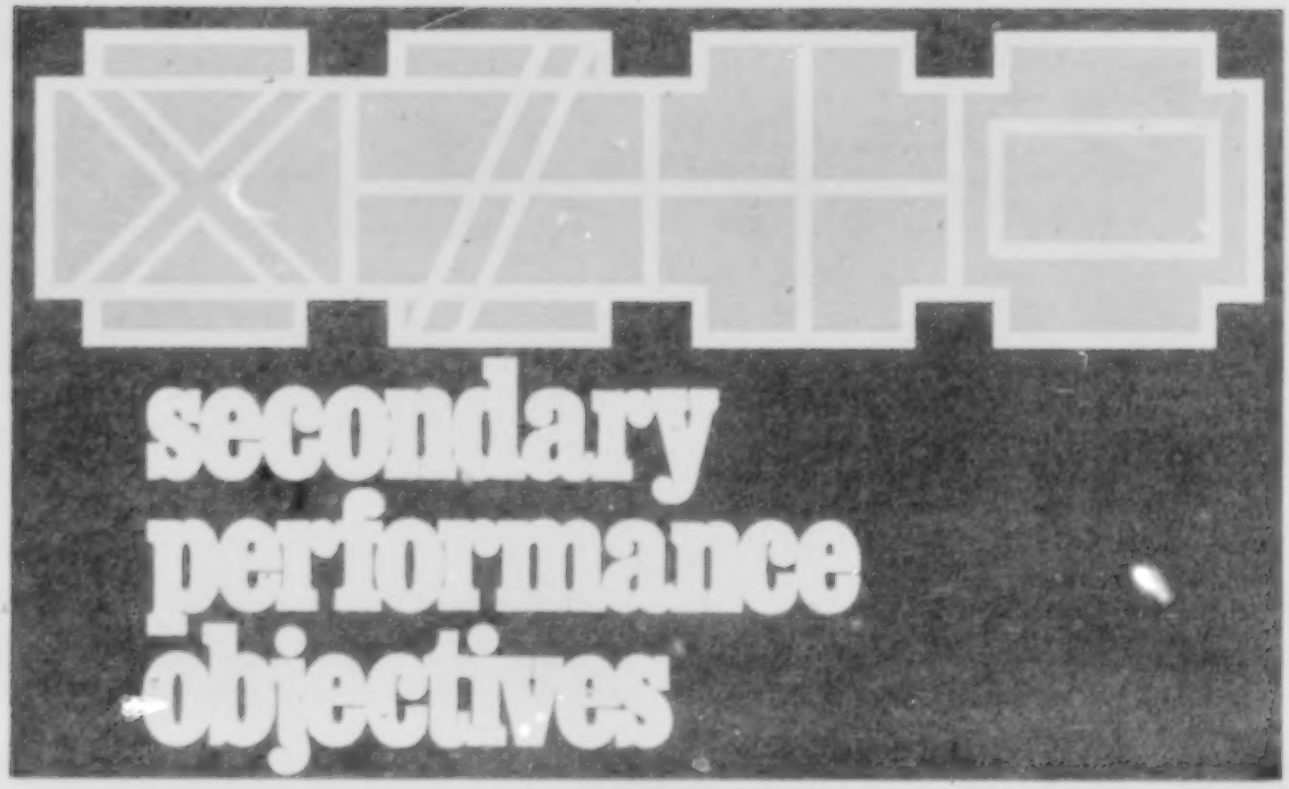
16. Read and interpret data given in circle graphs.

16.



The circle graph shows the incidence of each kind of book in the school library.

17. Construct a graph (bar, line, picture, circle) which represents a given set of numerical data.	17. Measure the hand spans (distance between the tip of little finger and thumb at widest stretch) of your classmates. Round off the measurements to the nearest centimetre and draw a bar graph to show how many of each size there are in the class.
18. Construct and interpret double-bar graphs.	18. Measure the hand spans and cubits (distance between longest finger tip and elbow) of your classmates. Round off the measurements to the nearest centimetre. Draw a double-bar graph, first showing measurements of hand spans, then cubits. Observe the relationships.
19. Count the number of groupings of a given size from a set of distinct objects.	19. How many different groups of two children can be made from a group of five children.
20. Count the number of finite arrangements for a set of "n" different objects.	20. a) How many ways can a set of four elements be arranged in a row? eight? n? b) How many arrangements can be made of three children taken two at a time.
21. Find the mean of a set of numbers.	21. From any telephone book, obtain the last digit of 25 telephone numbers. Find the average of the last digit and compare it with 4.5. Are all digits equally likely?
22. Find the various measures of central tendency of a given set of data.	22. A group of 10 children went out to trick-or-treat, decided to pool the candy they received, and to divide it among themselves. This is what they received: 32,28,28,32,32,12,10,22,17,17 candies The mean is 23. The mode is 32. The median is 25. Which of the three would be the fairest split?
23. Make a scatter diagram from a set of ordered pairs of data.	23. Measure the heights of your classmates in centimetres and record their ages in months. From the resulting ordered pairs (height, age) draw a scatter diagram showing these results. If there was a bunching at one corner of the diagram, how could it be redrawn to get the points spread out?
24. Select the most probable event from a finite sample and compute the numerical probability that each event will occur.	24. Given: a) A bag with five green and four red marbles, b) A bag with three green and three red marbles, c) A bag with five green, four red and six blue. In drawing a marble, which is most likely to be drawn in each of the examples a), b) and c). Compute each probability.
25. Analyze and solve basic probability problems.	25. In a toss of two coins, what is the probability of getting a head and a tail? In a toss of a set of dice, what is the probability of the sum of the numbers being eight?
26. Predict outcomes from multiple events.	26. Make a guess as to how many times four heads will appear in 16 tosses of four pennies. Then do the experiment.



Secondary Mathematics Courses

The Indiana General Commission on Education has approved the following course titles in mathematics for credit in grades nine through twelve.

Basic Mathematics
General Mathematics
Introductory Algebra
Algebra
Geometry
Informal Geometry
Advanced Mathematics (Unified)
Trigonometry
College Algebra
Analytic Geometry
Business Mathematics
Consumer Mathematics
Shop Mathematics
Probability and Statistics
Abstract Algebra
Computer Mathematics
Calculus

Each of these 17 named courses may be offered for one or more semesters of credit in multiples of 0.5 units. For example, most schools offer two units of algebra, but only one-half unit of analytic geometry.

Your attention also is called to the elimination of grade levels for the courses. Any course may be offered to students in any grade from nine to twelve, providing the prerequisites have been met. There is a logical sequence of courses, especially in the academic courses, which is well known to mathematics teachers. Flexibility is allowed so that a second year of algebra may precede or follow a course in geometry. These courses also could be taken concurrently.

One unit of mathematics is required for graduation in Indiana. Normally this unit consists of one full year of algebra, general mathematics or basic mathematics. Higher ability students may take algebra during the eighth grade. Use of this course for establishing high school graduation requirements shall be governed by Rule G-1 of the Commission on General Education. Students on a college preparatory curriculum should complete a minimum of two or three units of mathematics, preferably two units of algebra and one unit of geometry.

Mini-courses in mathematics also have been approved for graduation credit by the General Commission on Education. Mini-courses may be designed to meet specific objectives of mathematical learning on a wide variety of topics. Credit may be awarded in multiples of 0.05 unit or multiples of 1/12 unit. As an approximation, 1/12 unit of credit would correspond to three weeks of study on a special topic while 0.05 unit would be granted for 360 minutes of instructional time in a particular area. A student may accrue one unit of mini-course credit in mathematics to satisfy the graduation requirement. In order to offer mini-courses for credit, schools must submit a course outline and statement of objectives to the Commission on General Education.

The list of approved courses and the permissible

mini-courses provide great flexibility to schools in offering a broad curriculum to satisfy the highly variable mathematical needs of present day students. The goal in every school curriculum should be to *provide opportunities for four years of valuable mathematics for students of all interests and abilities.* For the students who begin the ninth grade with basic mathematics, general mathematics, algebra, geometry, or a mini-course, there should be various mathematics courses available for their selection in each succeeding year through the completion of their high school education. Students with lesser academic abilities or interests in mathematics should have viable courses available throughout their school careers. Likewise, students who have the potential for learning calculus in high school should not be denied that opportunity.

Following are some suggested sequences of courses which may be offered in Indiana schools. △

Grade	Normal Academic Sequence	Accelerated Academic Sequence	Sequence (a)	Sequence (b)
		(Algebra I in Grade 8)		
9	Algebra I	Geometry I*	General Math I	Intro. Algebra I
10	Geometry *	Algebra II*	General Math II	Intro. Algebra II*
11	Algebra II*	Advanced Math Computer Math	Algebra	Informal Geometry*
12	Advanced Math	Calculus	Geometry	Probability and Statistics

Grade	Sequence (c)	Sequence (d)	Sequence (e)	Sequence (f)
9	Basic Math	General Math I	General Math I	Algebra I
10	General Math I	Intro. Algebra I	Business Math	Geometry I
11	Intro. Algebra I	Informal Geometry*	Shop Math	Mini-Courses
12	Informal Geometry	Intro. Algebra II*	Consumer Math	Consumer Math

Many other variations are possible.

**may be interchanged.*

Objectives by Courses

Content of each course as indicated by the list of performance objectives is not limited to the stated objectives. Creative teachers will be able to extend the list of objectives according to the abilities and interests of students in the course. On the other hand, a course being taught under a stated title should aim toward successful completion of a great majority of the listed objectives. There is an overlap of objectives for certain courses, but the level of difficulty and depth of abstraction or application may vary substantially.

School marks (grades) should reflect the degree of the student's accomplishment in course objectives. For example, the objectives for Introductory Algebra I are less rigorous than similar objectives for Algebra I. The individual mark (grade) should be awarded for achievement of the objectives in each case.

Basic Mathematics

Upon completion of the course, the student should be able to do the following performance objectives:

1. Read and write decimal numerals for whole numbers to one billion.
2. Arrange whole numbers in order with or without a number line.
3. Compute sums, differences, products and quotients of whole numbers with reasonable accuracy.
4. Compute sums, differences, products and quotients of fractions with reasonable accuracy.
5. Compute sums, differences, products and quotients of decimals with reasonable accuracy.
6. Solve simple equations in one unknown.
7. Apply basic formulas to solve problems from business situations e.g., discounts, profit, etc.
8. Solve simply stated problems about personal finance, e.g., checking and savings accounts, budgets, loans, credit buying, etc.
9. Make measurements using a ruler in both customary units and metric units.
10. Calculate the perimeters and areas of simple plane figures.

11. Solve problems involving percents.
12. Apply the concept of ratio and proportion to scale drawings and problems derived from them.
13. Interpret and construct simple graphs about familiar relationships (circle, line and bar).
14. Perform basic operations on small integers.
15. Identify and construct or draw the basic plane figures, i.e., triangle, various quadrilaterals, other polygons, circles, parallel and perpendicular lines.
16. Apply mathematics to solve problems related to taxes (income, sales, property, excise, F.I.C.A.).
17. Apply mathematics to solve problems related to home improvement projects.
18. Calculate the median and arithmetic mean of a set of data; interpret the result.
19. Change units within the metric system.
20. Learn and apply ideas from number theory, e.g., prime numbers, GCF, LCM, divisibility tests, etc.

General Mathematics I (First Course)

Upon completion of the course, the student should be able to do the following performance objectives:

1. Compute with whole numbers up to 1,000,000 to find sums, differences, products, and quotients with reasonable speed and accuracy.
2. Compute with rational numbers in simple fraction form to find sums, differences, products and quotients with reasonable speed and accuracy. (at least to 100-ths).
3. Compute with rational numbers expressed as decimals (at least to 100-ths).
4. Convert common percents to fractions or decimal form and conversely.
5. a. Name the factors of composite whole numbers less than 100, and identify prime numbers.
   b. Apply the rules for order of operations on rational numbers.
6. Perform divisibility tests for 2, 3, 4, 5, 6, 9 and 10.
7. Find the greatest common factor, GCF, and least common multiple, LCM, of pairs of whole numbers each less than 100.
8. Recognize and use the basic properties of the numbers of arithmetic with respect to addition and multiplication, namely; closure, commutativity, associativity, distributivity, identity elements and inverses.*
9. Express relationships in the form of ratios, and write and solve problems involving proportions correctly describing realistic situations.
10. Solve simple problems up to and including three steps which involve applications of mathematics of geometry, business, home management, science and other situations relevant to present environment.
11. Prepare and interpret simple statistically oriented graphs of the following form: circle, bar, line and divided bar.
12. Identify and use non-metric relations using points, lines, and planes, specifically: coincidence, parallelism, perpendicularity and intersection.
13. Identify and use properties of simple plane figures including lines, line segments, rays, angles, triangles, quadrilaterals and circles.
14. Measure (using both customary and metric units) and compute lengths and areas using appropriate formulas for line segments, angles, polygons and other simple plane figures.
15. Construct, using ruler, compass and protractor, the common plane geometry figures mentioned above.
16. Explain the meaning and relationship of numbers in the set of integers.
17. Perform the operations of addition and subtraction on the set of integers with reasonable accuracy.
18. Use letters as variables in number relations or expressions.
19. Find numbers in a replacement set of a variable which makes a sentence involving the integers true.
20. Find and write the solution set of simple open sentences, primarily equations but including some inequalities.
21. Represent the solution sets of sentences in one variable on a number line.
22. Apply equation solving techniques to the solution of verbal problems, especially those involving formulas.
23. Find the absolute value of any integer.
24. Perform the basic operations with numbers expressed with exponents.
25. State and use the more common units of the metric system of measurement. Be able to work with these units.
26. Estimate answers to computational verbal problems.

General Mathematics II (Second Course)

Upon completion of the course, the student should be able to do the following performance objectives:

1. Compute sums, differences, products, and quotients using positive rational numbers between .001 and 1,000,000 written as whole numbers, fractions, or decimals.
2. Determine the order of rational numbers regardless

*Optional

of the written form.

3. Find the prime factors of composite numbers less than 500.
4. Find the exact positive square root of perfect square positive integers up to 1,000, and find the approximate square root of non-perfect square positive integers less than 1,000 using a table or by computation.
5. Solve open sentences (both equations and inequalities) involving first degree polynomials in one variable. Write the solution set involving rational numbers only, and check some of the numbers in the solution set.
6. Graph the solution set of equations and inequalities in one variable on a number line.
7. Solve for and check the solution set of simple systems of two linear equations in two variables.
8. From a list of formulas, select and use the formula that will aid in finding the solution to a realistic problem.
9. Write ratios and proportions to describe common relationships, and solve resulting equations when appropriate.
10. Use exponents to express powers of numbers, and perform operations with different powers of the same number.
11. State and apply the Pythagorean relations to problems involving right triangles.
12. Convert any common measurement to an equivalent measurement.
13. Measure in both customary and metric units, and use the results of measurements to calculate other measurements.
14. Construct and interpret graphs in two dimensions.
15. Write the definitions of the three basic trigonometric ratios in terms of right triangles and apply this knowledge to the solution of simple problems.
16. Draw "tree" diagrams, and calculate the probabilities of certain events.
17. Construct and analyze simple scale drawings.
18. Compute sums and products using integers whose absolute value is 500 or less.
19. Given a percent problem of the form a % of b = c, one of (a, b, c) is missing, find the missing part; use this knowledge to solve word problems involving percent.
20. Collect numerical data accurately and find the mean, median, mode, and range of the data.
21. Construct geometric figures and apply the fundamental properties of congruence and similarity.
22. Determine lines and points of symmetry in geometric figures and other objects.

Introductory Algebra I (First Course)

Note: Introductory Algebra I and Introductory Algebra II are two full-year courses which encompass the essential objectives of Algebra I. The level of difficulty of problems and the degree of performance expected are somewhat less for Introductory Algebra than for Algebra I.

Upon completion of the course, the student should be able to do the following performance objectives:

1. Perform with reasonable speed and accuracy the basic operations of arithmetic on a set of non-negative rational numbers expressed in fraction or decimal form.
2. Write algebraic expressions or sentences which represent written or verbally stated conditions.
3. Evaluate simple algebraic expressions given replacement values of variable(s).
4. Use exponents to represent whole number powers of a given base — write exponent expressions as the product of factors.
5. Use the properties of exponents to simplify monomial expressions involving powers.
6. Find the solution set and check the root when given a simple equation in one variable.
7. Find the solution set and check representative numbers in it when given a simple inequality in one variable.
8. Represent (graph) the solution set of selected sentences (equations or inequalities) on the number line (see objectives five and six).
9. Apply the order of operations conventions to a series of indicated operations on numbers.
10. Solve verbal problems which may be represented by simple equations in one variable.
11. Use ratio and proportion to solve problems which involve these concepts.
12. Graph a point on a coordinate plane when given the coordinates of the point; state the coordinates of a given point on the coordinate plane.
13. Represent the graph of a simple linear equation in two variables as a line on the coordinate plane.
14. Given two simple polynomials, find their sum or difference.
15. Multiply a given polynomial by a monomial or binomial.
16. Divide a given polynomial by a monomial.
17. Find the prime factorization of a given composite number.
18. Solve problems involving percent, e.g., discount, taxes, commission.
19. Given a problem which may be solved by applying a formula, select the appropriate formula, substitute correct values, solve the resulting equation

and check the answer.

20. Perform the basic arithmetic operations on the set of integers (directed numbers).

21. Measure in both customary and metric system and apply the results to solve problems, e.g., perimeter, area, volume.

Introductory Algebra II (Second Course)

Note: Introductory Algebra I and Introductory Algebra II are two full-year courses which encompass the essential objectives of Algebra I. The level of difficulty of problems and the degree of performance expected are somewhat less for Introductory Algebra than for Algebra I.

Upon completion of the course, the student should be able to do the following performance objectives:

1. Demonstrate satisfactory performance on objectives stated under Introductory Algebra I.
2. Interpret, write and use number expressions involving absolute value.
3. Perform arithmetic operations on the set of all rational numbers.
4. Graph sets of numbers on the number line.
5. Graph ordered pairs of real numbers on the coordinate plane.
6. Solve equations more complex than those required in Introductory Algebra I by transforming sentences into equivalent sentences using basic principles.
7. Add, subtract, multiply and divide polynomials of medium complexity — raise to the second or third power.
8. Know and apply the laws of exponents (for non-negative integers).
9. Factor polynomials which can be expressed as the product of a monomial and a polynomial or as the product of a simple binomial and a polynomial.
10. Know and use the special products and factors for simple polynomials e.g., binomial squared, differences of squares, etc.
11. Solve quadratic equations which can be factored into the product of two simple linear factors.
12. Use the quadratic formula to solve a general, single variable quadratic equation.
13. Add, subtract, multiply, divide and simplify simple rational expressions (fractions which contain variables).
14. Solve systems of two linear equations in two variables and verify the common solution if it exists.
15. Graph a system of two linear equations or inequalities in two variables on a coordinate system — interpret the graph.
16. Find the approximate square root of a number less than 1,000 by means of a table and/or an algorithmic computation procedure.
17. Simplify expressions containing radicals.
18. Apply algebraic techniques to the solution of a variety of simple verbal problems.
19. Use formulas to solve problems from science, business and geometry.
20. Identify functions and distinguish from non-functional relations.
21. Given the equation of a line or the graph of a line, determine its slope.
22. Given the slope and a point, graph the line that has the given slope passing through the given point.

Algebra I (First Course)

Upon completion of the course, the student should be able to do the following performance objectives:

1. Graph a set of real numbers on the number line.
2. Identify and state order relations on the set of real numbers.
3. Perform the operation of addition, subtraction, multiplication and division on the set of rational numbers.
4. State and use the basic properties of addition and multiplication on the real number system, namely: Closure, commutativity, associativity, distributivity, identity elements and inverses.
5. Apply the "order of operations" to a series of indicated arithmetic operations.
6. Evaluate algebraic expressions by direct substitution.
7. Solve open linear sentences (equalities and inequalities), write and graph solution sets.
8. Graph the solution sets of open sentences on number lines.
9. Identify the domain and range of a relation or function.
10. Solve linear equations that contain more than one operation, and justify each step.
11. Develop other properties of number sets (odd, even, primes, etc.) and prove *selected theorems about operations on numbers.
12. Analyze verbal problems, translate into mathematical sentences, find the solution set and check.
13. Graph linear equations with two unknowns on a Cartesian coordinate plane.
14. Find the slope of a line given its equation or two points on it.

*Optional

15. Find by graphing the approximate coordinates of the point, if it exists, in the solution set of a system of two linear equations in two variables.
16. Find the distance between two points on a coordinate system.
17. State and apply the properties of integer (and rational*) exponents.
18. Perform the basic operations on polynomials.
19. Find the common solution of a system of linear equations by substitution and addition methods.
20. Find the special products of polynomials and the special factors of polynomials.
21. Determine the factorability of a general trinomial, and if it is factorable, determine its factors.
22. Solve quadratic equations with real roots by factoring, completing the square, and using the quadratic formula.
23. Determine whether a quadratic equation has real or imaginary roots without solving the equation.
24. Extend the fundamental operations to include rational expressions formed by quotients of polynomials.
25. Write and solve equations involving proportions.
26. Find the approximate square root of a number by an algorithm or by a table.
27. Perform the basic operations on the set of irrational numbers expressed as radicals. Rationalize numerators and denominators, and express them in simplest radical form.
28. Distinguish between relations which are functions and those which are not functions.
29. Solve problems which relate to simple probability.*
30. Solve problems which relate to trigonometric relations.*
31. Use set terminology and perform basic operations on sets.*

Algebra II (Second Course)

Upon completion of the course, the student should be able to do the following performance objectives:

1. State and illustrate the basic properties of important number sets (integers, rational numbers, real numbers, complex numbers).
2. Perform computations on the set of real numbers (emphasis on the integers and rational numbers).
3. Perform the fundamental operations on polynomials and simple rational expressions.
4. Solve and check the solution to equations in one variable.
5. Solve and graph the solution set to inequalities in one variable.
6. Solve a two x two system of equations by various methods.
7. Given two equations in two variables, construct the graph and interpret the results.
8. Factor polynomials and rational expressions in one or more variables.
9. Solve three x three and higher systems of equations.
10. Distinguish between relations and functions. Construct the graphs of each and interpret the results.
11. Prove selected theorems pertaining to algebra, e.g., factor theorem.
12. Construct the inverse of a function, and determine whether it is a function.
13. Recognize and construct special functions; constant, absolute value, greatest integer, etc.
14. Determine the distance between two points in a plane, and find the coordinates of the segment's midpoint joining the points.
15. Given sufficient conditions, write the equation of a line using standard forms: intercept, two-point, etc.
16. Solve quadratic equations by various methods.
17. Graph a given quadratic function, determine its domain and range, find the zeros, locate axis of symmetry, and determine absolute maximum (minimum) values.
18. Relate the sum and difference of the roots of a quadratic equation to the coefficients, and determine their number and nature.
19. Solve and graph selected linear inequalities.
20. Solve and graph selected quadratic inequalities.
21. Write and solve equations involving direct and inverse variation.
22. Identify and graph second degree equations that result in parabolas, hyperbolas, circles, ellipses.
23. Derive the equations of the central conics.
24. Solve problems involving conic sections including translations.
25. Solve selected exponential equations, and graph selected exponential functions.
26. Use the fundamental laws of logarithms, compute selected arithmetic expressions, solve simple logarithmic equations, and graph selected logarithmic functions.
27. Develop and use formulas related to sequences and series.
28. Solve problems involving metric units of measure.
29. Identify and define the trigonometric (circular) functions.*
30. Graph the trigonometric (circular) functions.*
31. Develop and use trigonometric identities.*

*Optional

32. Solve triangles requiring trigonometry.*
33. Apply trigonometry to practical situations.*
34. Identify and use basic theorems on probability.*
35. Define and perform basic operations upon matrices.*
36. Use matrices to solve systems of equations.*

Informal Geometry

Note: This course is intended primarily for students whose interests and objectives lie in applications of geometry to practical fields. The geometrical content should be presented at a challenging level but the emphasis on formal proof should be greatly reduced or eliminated while logical development is retained. Practical applications of geometry to real world situations is a primary goal of the course.

Upon completion of the course, students should be able to do the following performance objectives:

1. Apply the concepts of union and intersection of sets of geometric figures.
2. Recognize and use the concept of inductive reasoning as it applies to the development of geometric facts.
3. Demonstrate an understanding of the concepts of point, line, line segment, plane and solid figures; read and use appropriate notation.
4. Demonstrate an understanding of angle, the naming of angles, special angles; develop skill in measuring angles with the protractor.
5. Demonstrate an understanding of congruence as applied to line segments, angles, and geometric figures.
6. Draw (construct) the basic constructions using compass and straight edge and use these constructions to complete more complex figures.
7. Draw straight line geometric figures using ruler and protractor.
8. Recognize and use parallel and perpendicular lines with associated facts regarding the angles formed.
9. Recognize various triangles by shape and recognize and use their associated line segments such as altitudes, angle bisectors, and perpendicular bisectors of sides.
10. Recognize and use key facts (standard theorems) about the common quadrilaterals such as rectangles, parallelograms, rhombuses, squares, and trapezoids.
11. Visualize lines and planes in three dimensions and make descriptive sketches on a plane.
12. Find areas and perimeters of triangles, rectangles, squares, trapezoids and regular polygons; develop the respective formulas and apply to practical problems involving linear and square units.
13. Recognize and use circles and spheres with related parts such as radius, diameter, arc, chord, tangent, secant and sector.
14. Compute circumferences and areas of circles, and solve practical problems involving these measurements.
15. Be able to use the Pythagorean theorem in applications to right triangles and figures reducible to right triangles.
16. Compute volumes of common solids and apply to problems about capacity and weight.
17. Compute lateral and total surface areas of common solids and apply to practical problems.
18. Demonstrate a knowledge of similarity and similar figures; use ratios and proportions to solve for missing parts; apply to maps and scale drawings.
19. Recognize and use trigonometric ratios of sine, cosine, and tangent as related to the right triangle; apply to practical problems involving right triangles.
20. State and use the special properties of equilateral triangles.
21. Describe and locate by construction methods, the incenter, circumcenter, centroid, and orthocenter of triangles.
22. Recognize and illustrate dihedral and polyhedral angles.
23. Read and state with understanding selected proofs of key geometric theorems; develop simple original proofs.*
24. Make measurements of objects in both metric and customary units, and compute areas and volumes from the data; apply to practical problems.
25. Solve formulas about geometric figures for any one variable when given the replacement for all other variables in the formula.
26. Find approximate areas, perimeters, or volumes of irregularly shaped figures.

Geometry

Upon completion of the course, the student should be able to do the following performance objectives:

1. Apply the concept of set notation, including subsets and the operations of union and intersection, to geometric figures.
2. Use the basic axioms and theorems regarding operations on real numbers.
3. Identify the components of a deductive system: Undefined terms, defined terms, assumptions (axioms or postulates) and theorems.
4. Recognize and distinguish between deductive and

*Optional

inductive reasoning and their relationship to daily living.

5. Define basic geometric figures such as line segment, skew lines, parallel lines, ray, angle, triangle, parallelogram, trapezoid, square, rectangle, regular polygon, etc.
6. Recognize and use concepts of elementary logic to formulate proof of a theorem.
7. State the meaning of congruence as applied to line segments, angles, triangles and other geometric figures.
8. Write simple proofs applying the basic congruence properties for congruent triangles. (ASA, SAS, SSS)
9. Write simple proofs applying the basic theorems specifically applicable to right triangles.
10. State and apply methods of proving lines and planes parallel.
11. State and use conclusions which follow from having lines and/or planes parallel.
12. Recognize and use key facts (standard theorems) about the common quadrilaterals such as rectangles, parallelograms, rhombuses, squares and trapezoids.
13. Define similarity for geometric figures, prove similarity theorems, and use subsequent properties to solve problems.
14. Work numerical problems using ratios and proportions, including the geometric mean, related to similar polygons.
15. State, prove and use the Pythagorean theorem in proofs and in applications.
16. Define circle, sphere, and their related parts: Diameter, radius, arc, chord, tangent, secant, segment and sector.
17. Use postulates and theorems pertaining to congruent radii, chords, central angles, and arcs in writing proofs concerning circles.
18. Find areas and perimeters of triangles, trapezoids, parallelograms, rectangles, squares, circles, and regular hexagons and octagons, and geometric figures which are a composition of more than one figure.
19. Find volumes of spheres, cones, cylinders, prisms and pyramids.
20. Recognize and write indirect proofs.
21. Make simple constructions with compass and straightedge. These should include copying a line segment, copying an angle, the perpendicular bisector of a line segment, bisecting an angle, the perpendicular to a line through a given point, a line parallel to a given line through a point not on the given line, dividing a given segment into a specified number of congruent segments.
22. Use a protractor to measure angles.
23. State and use the special properties of isosceles triangles.
24. State and use special properties of equilateral triangles.
25. Describe and locate by construction the incenter, circumcenter, centroid, and orthocenter of triangles.
26. Characterize and locate specific sets of points, such as: Points a given distance from a given point, points a given distance from a given line, points equidistant from the sides of a given angle.
27. Find the surface area of spheres and the total and lateral areas of cones, cylinders, prisms and pyramids.
28. Use the coordinate geometry to describe points and lines in a plane.
29. State and use the basic inequalities relating to sides and angles of triangles.
30. State the converse, inverse, and contrapositive of a theorem, and identify equivalent statements.
31. Recognize and illustrate dihedral and polyhedral angles.
32. Determine the sum of the interior angles of any polygon, and, if the polygon is regular, determine the measure of each interior and exterior angle.
33. Determine the measures of appropriate line segments, angles and arcs in circles intersected by chords, diameters, secants and tangents.
34. For any quadrilateral given sufficient data: a) Classify it according to type; b) Determine the measures of other segments; and c) Determine the existence of parallelism and perpendicularity.
35. Give a counterexample to prove a statement false.
36. Determine symmetries with respect to geometric figures.
37. Work problems of geometric application involving metric units of measure.
38. Prepare or explain proofs of theorems involving coordinate geometry.*

*Optional

Advanced Mathematics (Unified Course)

Note: This one-year course may be considered equivalent to a combination of topics from the courses in Trigonometry, College Algebra and Analytic Geometry.

Upon completion of the course the student should be able to do the following performance objectives:

1. Identify and use the field properties on the set of real numbers.
2. Prove selected theorems concerning a set of real numbers and major subsets.
3. Coordinate a line and a plane; graph selected sets of real numbers and ordered pairs of real numbers.

4. Define and make use of notation for functions.
5. Identify and solve problems about functions, e.g., absolute value, inverses, composition.
6. Graph a variety of special functions.
7. Determine domain and range of functions.
8. Develop theorems about polynomials and solve problems involving such functions, e.g., maxima, zeros, etc.
9. State and use the distance formula.
10. Write the defining equation for the exponential function, state the domain and range, and graph the exponential function for various permissible values of the base.
11. Write the defining logarithmic equation equivalent to a given exponential function, state the domain and range, and graph the logarithmic function for various permissible values of the base.
12. Solve exponential and logarithmic equations, and apply to the solution of problems.
13. Solve problems involving direct, inverse, and joint variation.
14. Use synthetic division to divide a polynomial P(x) by a polynomial of the form x + a.
15. State, prove, and use the remainder, factor, and converse of factor theorems.
16. Indicate the number of roots of real polynomial equations, and find real zeros and solution sets respectively.
17. Use approximation techniques to find real zeros of polynomials with real coefficients.
18. State the definition of the six circular (trigonometric) functions derived from the "wrapping" function on the unit circle.
19. Derive, state and use the fundamental trigonometric relations in proving trigonometric identities.
20. Convert degree measure to radian measure of an angle and vice versa.
21. Find, using tables, the value of any circular (trigonometric) function given the argument in degrees or radians.
22. Solve selected trigonometric equations.
23. Sketch the graphs, over the interval $(-2\pi, 2\pi)$ for all circular (trigonometric) functions.
24. State the period, amplitude and phase shift of circular functions as they apply.
25. Define, state and use the trigonometric formulas for sum, difference, double and half angles.
26. Define and graph the inverses of the circular functions. Limit the domain of the inverses so that they are functions, and evaluate expressions involving these.
27. Solve triangles using law of sines and law of cosines.
28. State and use the rule of equality of complex numbers, and perform the four fundamental operations on complex numbers in either a + bi form or (a, b) form.
29. State the principle of mathematical induction, and prove selected theorems by using mathematical induction.
30. Construct the polar coordinate system in a plane, and determine the polar coordinates of any point.
31. Give both the rectangular and polar form of a complex number. (Include exponential form).
32. Find the product and quotient of two complex numbers using the polar form relations.
33. State and use DeMoivre's Theorem. *(Prove the theorem by mathematical induction).
34. Calculate the nth roots of a complex number.
35. Use the binomial theorem to write the expansion of $(a + b)^n$ where n is a positive integer and write the nth term of the expansion. *(Prove the binomial theorem).
36. Use the binomial theorem to write the first k terms in the expansion of a binomial when n is a non-integer rational number.
37. Determine P (n, r) for $r \leq n$, circular permutations, and C (n, r) for $r \leq n$.
38. Determine the probability of success of a single trial if there are n equally likely outcomes.
39. Determine the probability of occurrence of two or more independent events, dependent events, and mutually exclusively events.
40. Determine the probability of an event happening exactly r times in n trials, and at least r times in n trials.
41. Define the following: sequence, series, arithmetic series (sequence), geometric series (sequence), harmonic series (sequence).
42. Write the terms of a sequence from the appropriate recursion formula.
43. Determine the nth term of selected arithmetic and geometric sequences, and determine the sum of a finite number of terms of selected arithmetic and geometric series. Find the sum of convergent infinite geometric series.
44. Define matrix; define vector and determinant in terms of a matrix.
45. Calculate the sum of two vectors, the product of a scalar and a vector, the inner (dot) and outer products of two vectors, and the norm (length) of a vector. *Graph these operations on vectors on the complex plane.
46. State and illustrate the properties of the algebra of two-space and three-space vectors.

*Optional

47. Find the sum of two matrices, the product of a scalar and a matrix, and the product of two matrices.
48. Evaluate second, third and fourth order determinants by using the properties of equivalent determinants and/or expansion by minors.
49. Define and use the properties of a line given the defining equation.
50. Define and illustrate the conic sections (circle, ellipse, parabola, hyperbola and degenerate cases).
51. Define and use the special points and lines associated with the conics (focus, directrix, major axis, eccentricity, asymptotes, etc).
52. Simplify equations of conics by translation and rotation of axes.
53. Sketch the graph of any equation of the form $Ax^2 + Cy^2 + Dx + Ey + F = 0$ (A, C, D, E and F are real numbers and not both A and C equal 0).
54. Prove theorems involving points, lines and the conics.
55. Write the equation of a conic, given sufficient constraints.
56. Apply the relations about conics to the solution of problems.
57. Write equations of lines and planes in three-space.

Trigonometry

Note: The essential elements of this course are incorporated in the objectives for Advanced Mathematics [Unified].

Upon completion of this course, the student should be able to do the following performance objectives:

1. Write the definitions of the circular (trigonometric) functions.
2. Develop and use fundamental relationships (identities) among the circular functions.
3. Verify identities by referring to fundamental relationships.
4. State the period, amplitude and phase shift (as applicable) of the circular (trigonometric) functions.
5. Construct and interpret graphs of the circular functions.
6. Find, using tables, the value of any circular function for any permissible real argument.
7. State, prove and use formulas for sum, difference, half and double angles for any circular function.
8. Define and develop relations for the inverses of the circular functions.
9. Limit domains and ranges so that the inverses are functions; sketch the graph of the functions and inverse within specified domains.
10. Apply the circular functions to solve problems of uniform circular and harmonic motions.
11. Develop and apply the law of sines and law of cosines to solve triangles.
12. Solve practical problems involving use of law of sines and law of cosines.
13. State the properties of vectors and perform the operations involving vectors.
14. Solve problems using vectors.
15. Relate polar coordinates to Cartesian coordinates.
16. Write complex numbers in polar or rectangular form.
17. State, prove and use DeMoivre's theorem to find products, quotients, powers and roots of complex numbers.
18. Solve trigonometric equations for principal and general values.

College Algebra

Note: The essential elements of the course are incorporated in the objectives for Advanced Mathematics [Unified].

Upon completion of this course, the student should be able to do the following performance objectives:

1. Identify and use the field properties on the set of real numbers.
2. Prove selected theorems concerning the set of real numbers and major subsets.
3. Coordinate a line and a plane; graph selected sets of real numbers and ordered pairs of real numbers.
4. Define and make use of notation for functions.
5. Identify and solve problems about functions, e.g., absolute value, inverse, composition.
6. Graph a variety of special functions.
7. Determine domain and range of functions.
8. Develop theorems about polynomials and solve problems involving such functions, e.g., maxima, zeros, etc.
9. Write the defining equation for the exponential function, state the domain and range, and graph the exponential function for various permissible values of the base.
10. Write the defining logarithmic equation equivalent to a given exponential function, state the domain and range, and graph the logarithmic function for various permissible values of the base.
11. Solve exponential and logarithmic equations, and apply to the solution of problems.
12. Solve problems involving direct, inverse and joint variation.
13. Use synthetic division to divide a polynomial P(x)

*Optional

by a polynomial of the form x + a.

14. State, prove and use the remainder, factor, and converse of factor theorems.
15. Distinguish between polynomials and polynomial equations, and find the zeros and solution sets respectively.
16. Indicate the number of roots of real polynomial equations, and form real polynomial equations using given roots.
17. Determine the rational roots of polynomial equations.
18. Approximate real zeros of polynomials with real coefficients.
19. State the Principle of Mathematical Induction, and prove selected theorems by using mathematical induction.
20. Solve equations involving the properties and operations of complex numbers.
21. Use the binomial theorem to write the expansion of $(a + b)^n$ where n is a positive integer, and write the rth term of the expansion. (*Prove the binomial theorem.)
22. Use the binomial theorem to write the first k terms in the expansion of a binomial when n is a non-integer rational number.
23. Determine P (n, r) for $r \leq n$, circular permutation, and C (n, r) for $r \leq n$.
24. Determine the probability of success of a single trial if there are n equally likely outcomes.
25. Determine the probability of occurences of two or more independent events, dependent events, and mutually exclusively events.
26. Determine the probability of an event happening exactly r times in n trials, and at least r times in n trials.
27. Define the following: Sequence, series, arithmetic series (sequence), geometric series (sequence), harmonic series (sequence).
28. Write the terms of a sequence from the appropriate recursion formula.
29. Determine the nth term of selected arithmetic and geometric sequences, and determine the sum of a finite number of terms of selected arithmetic and geometric series. Find the sum of convergent infinite geometric series.
30. Define matrix; define vector and determinant in terms of a matrix.
31. Calculate the sum of two vectors, the product of a scalar and a vector, the inner (dot) and outer products of two vectors, and the norm (length) of a vector. Graph on the complex plane.
32. Find the sum of two matrices, the product of a scalar and a matrix, and the product of two matrices.
33. Evaluate second, third and fourth order determinants by using the properties of equivalent determinants and/or expansion by minors.
34. Solve systems of equation using matrices, determinants in addition to algebraic methods.

Analytic Geometry

Upon completion of the course, the student should be able to do the following performance objectives:

1. Utilize Cartesian Coordinate system for naming points in a plane.
2. Compute the distance between any two points in a plane.
3. Determine the coordinates of a point that divides a line segment into parts whose ratio is given.
4. Determine the slope of a line and the related angle of inclination.
5. Find the slope of a line perpendicular to a given line.
6. Determine the equation of a line which a) is parallel to either axis, b) passes through two given points, c) passes through a given point and has a given slope.
7. Associate a given line with an equation of the form $Ax + By + C = 0$ (A, B and C real numbers and not both A and B equal 0).
8. Determine a specified member from a given family of lines.
9. Determine a) existing intercepts, b) symmetry relative to axes and origin, c) horizontal and vertical asymptotes, d) excluded regions, given an equation expressed in rectangular or polar coordinates.
10. Discuss the nature of the graph of $y = ax^n$ for replacements of a by real numbers and replacements of n by rational numbers.
11. Translate the origin to a new position (a, b) in the plane by using the translation equations $x = x^1 + a$ and $y = y^1 + b$.
12. Sketch the graph of any equation of the form $Ax^2 + By^2 + Cx + Dy + E = 0$ (A, B, C, D and E real numbers and not both A and B equal 0).
13. Describe each conic section as the intersection of a plane and a conical surface.
14. Use the general locus definitions to derive the equations of each of the conic sections and define associated terms (e.g., directrix, focus, eccentricity, asymptotes).
15. Rotate a given coordinate axis system through a given angle.
16. Determine which of the conic sections is represented by the general equation of the conic $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, given the constants.
17. Rotate the axes to remove the product term from

the general equation of the conic.

18. Recognize equations of conics in polar coordinate form.
19. Write the parametric equations of a line.
20. Sketch the graph of a function expressed by parametric equations.
21. Use a three-dimensional rectangular coordinate system to name and locate points in space.
22. Sketch the graph of cylinders, cones and quadric surfaces.
23. Sketch the graph of the intersection of two surfaces.
24. Define a surface of revolution.
25. Apply analytical methods to the proofs of theorems.
26. Compute the distance between two points in space.
27. Find the direction cosines and direction numbers of a directed line.
28. Find the distance from a point to a plane or a line.
29. Use vector products to solve problems.

Business Mathematics

On completion of the course, the student should be able to do the following performance objectives:

1. Perform the four basic operations with whole numbers, fractions, and decimals with reasonable speed and accuracy.
2. Compute mental multiplications and divisions by 10, 100, and 1000, and the costs of articles purchased in quantities of 10, 100, and 1000; articles priced at 10 cents, $1, $10, etc.; articles priced per 10, 100, and 1000.
3. Reconcile a bank balance, personal or business; complete sales slips, including computation of a sales tax and a federal excise tax; and compute aliquot parts, with reasonable accuracy.
4. Determine wage income from hourly wages consisting of straight time, overtime, and deductions, including income tax, FICA tax, and miscellaneous deductions.
5. Recognize and apply percentage relationships in solving problems.
6. Determine income from commissions and commissions plus salary.
7. Compute interest using tables and using formulas.
8. Calculate the interest charges and determine the effective rate of interest on installment purchases.
9. Determine costs of life, automobile and home insurance.
10. Read utility meters and determine utility bills.
11. Figure costs of shipping goods by parcel post, express and freight.
12. Compute property, federal income and state income taxes.
13. Prepare and use charts that picture the results of business operations.
14. Draw, read and interpret circle, bar, picture and line graphs.
15. Compute trade, single rate and cash discounts.
16. Compute depreciation by the straight line, declining balance, and sum-of-the-years-digits methods.
17. Convert units within the customary and metric systems.
18. Convert units between the customary and metric systems of measure (using tables or calculators).

Consumer Mathematics

Note: This course may be taught at varying levels of difficulty. The basic objectives should be retained, but the depth and complexity of application problems may be adjusted to the level of student's ability and mathematical background. Grouping by class level is suggested, or intra class grouping may be used to adjust the content and depth. The course may extend over one or two semesters.

Upon completion of this course the student should be able to do the following performance objectives:

1. Perform the basic arithmetic operations on nonnegative rational numbers in whole number, fraction or decimal form.
2. Perform the calculations involved in checking a typical paycheck stub, e.g., gross amount, rates, tax deductions, FICA, insurance, dues, and net amount.
3. Perform the calculations involved in budgeting personal finances, e.g., income from various sources, installment payments, basic necessities of food, clothing, shelter, savings, and miscellaneous expenditures.
4. Perform the calculations necessary for operation and maintenance of a typical home or apartment, e.g., rent or payments, utility bills, repair costs, maintenance cost, insurance, service costs, etc.
5. Perform the calculations necessary for paying taxes, e.g., sales tax, federal and state income taxes, real property taxes, personal property taxes, excise taxes, licenses, etc.
6. Perform the calculations involved in Social Security payments and benefits, also retirement plans and other forms of life insurance.
7. Perform the calculations necessary for figuring the cost of purchase, maintenance, and insurance on a car, e.g., payment schedules, types and amount of car insurance, maintenance costs, repairs, operating costs for gas and oil, fuel consumption, travel records, etc.
8. Perform the calculations necessary for operating types of investments, e.g., bank accounts, certifi-

cates of deposit, stocks, bonds, mutual funds, etc.

9. Perform the calculations necessary for operating a small business, e.g., inventories, purchases, rent, taxes, labor, utilities, loans, discounts, commissions, etc.
10. Read, interpret and draw various types of graphs, e.g., circle, line, bar, divided bar.
11. Perform the calculations related to savings and checking accounts — reconciling record service charges, interest, etc.
12. Perform the calculations involved in insurance of all types — liability, health and accident, life, fire, homeowners, special purposes, etc.
13. Measure and use measurements of length, area, volume, capacity, weight (mass), temperature and time in both customary and metric units.
14. Solve problems using formulas from business, science, geometry and other practical areas.
15. Read tables to find amounts for installment payments, interest, mortgage payments, taxes, mortality, powers and roots, trigonometric values, airline, rail and bus schedules, etc.
16. Compare price advantages of sales, discounts, etc. by calculating unit prices where appropriate, or total costs including finance charges where appropriate.
17. Read and interpret financial reports and graphs.
18. Read, interpret and use various market reports.
19. Read, interpret and use data about sports.
20. Apply mathematical techniques to the cost and value of "do-it-yourself" projects.
21. Solve problems which involve large numbers such as those that deal with total population or consumer impact.
22. Conduct experiments and collect data as a basis for solving problems or interpreting common uses of mathematics.

Shop Mathematics

On completion of the course, the student should be able to do the following performance objectives:

1. Perform the fundamental operations on integers, rational numbers and real numbers.
2. Make precise measurements with inch and centimetre rulers.
3. Read and interpret bar and line graphs.
4. Work with dimensions expressed in mixed measurements, e.g., in feet and inches or in metres and centimetres.
5. Make industrial calculations involving fractions, decimals and estimation.
6. Use percents to determine discounts, tolerances and efficiencies.
7. Convert measures from one unit to another in the customary system.
8. Convert measures from one unit to another in the metric system.
9. Convert measures between the customary and metric systems (using tables or calculators).
10. Convert temperature measures between the Celsius and Fahrenheit systems (using tables or calculators).
11. Find the perimeters of rectangles and the circumferences of circles.
12. Find the areas of parallelograms, triangles, trapezoids and circles.
13. Find the surface areas of cylinders, cones and spheres.
14. Find the volumes of rectangular solids, cylinders, cones, pyramids and spheres.
15. Use formulas to solve typical industrial problems.
16. Solve linear equations using the addition, subtraction, multiplication and division rules.
17. Apply the equation-solving process to industrial formulas.
18. Perform the standard geometric constructions.
19. Apply geometric constructions to shop and drawing problems.
20. Use similar and congruent triangles to solve problems involving right triangles.
21. Use the Pythagorean relationship to solve problems involving right triangles.
22. Use ratios to compare quantities.
23. Find the missing term in a proportion.
24. Use proportions to find lengths, areas and volumes of similar objects.
25. Estimate the costs of shop projects.
26. Determine quantities of materials needed for shop projects.
27. Solve power and energy related problems.*
28. Solve problems related to particular shop areas, like the construction, machine and printing trades.*

Probability and Statistics

Upon completion of the course, the student should be able to do the following performance objectives:

1. Interpret data presented in tabular and graphical form.
2. Organize and present data in tabular and graphical form. For example: bar graph, broken line graph, circle graph, pictorial chart, histogram, frequency polygon, and cumulative frequency polygon.
3. Summarize and analyze selected data by calculat-

*Optional

ing the measures of central tendency (mean, median, mode); discuss their relative merits.

4. Calculate measures of dispersions: range, quartile deviation, variance, and standard deviation.
5. Compute measures of central tendency and measures of dispersion from ungrouped data.
6. Compute the mean and standard deviation for large numbers of measurements by grouping techniques.
7. Determine whether outcomes of an experiment are equally likely.
8. Determine whether outcomes of an experiment are mutually exclusive.
9. Determine the sample spaces of various experiments.
10. Express events as subsets of a sample space.
11. Determine whether two events are complementary.
12. Determine whether events are exhaustive.
13. Develop and apply the formulas for calculating the number of permutations and combinations of n objects taken r at a time, and determine whether the problem involves permutations or combinations.
14. Apply correctly the two basic counting principles to determine the number of outcomes in event "A or B" and event "A and B."
15. Determine the probability of an event in a finite sample space using the classical definition of theoretical probability.
16. Estimate the probability of an event by the relative frequency concept in a series of experiments.
17. Apply combinatorial theory to calculate the probability of events.
18. Identify binomial experiments and apply the laws of chance to the binomial distribution.
19. Compute the conditional probability of event A given B.
20. Determine whether two events are independent.
21. Relate the probability of occurrence of two independent events to the probability of their intersection and apply to appropriate problems.
22. Compute the mathematical expectation of an event from an actual or theoretical experiment.
23. Apply the appropriate additive and multiplicative theorems to determine the probability of multiple events.
24. Distinguish between samples and populations and between statistics and parameters.
25. Use elementary sampling theory to perform a random sampling.
26. Apply the theory of probability to acceptance sampling.
27. Apply the theory of probability to test statistical hypotheses involving normal and binomial distributions.
28. Use samples to make estimates of population measures.

Abstract Algebra

Upon completion of the course, the student should be able to do the following performance objectives:

1. Describe finite and infinite sets using roster or rule notation.
2. Perform the operations of union, intersection, complementation on sets.
3. Determine a product set of two given sets.
4. Prove the basic theorems on union and intersection of sets.
5. Illustrate relations, mappings and functions as subsets of a product set.
6. Define an equivalence relation and determine which of the properties of an equivalence relation that a given relation exhibits.
7. Determine the equivalence classes associated with a given equivalence relation.
8. Define a binary operation in terms of a mapping from a product set to a set.
9. Define groups and semi-groups and prove the basic group theorems.
10. Determine whether or not a given structure fits the group properties.
11. Define subgroups and determine whether a given structure fits the conditions of being a subgroup.
12. Define rings and integral domains and prove the basic theorems associated with these structures.
13. Define subrings and determine whether a given structure fits the conditions for being a subring.
14. Define field and prove the basic theorems on fields.
15. Determine whether or not a given structure fits the field properties.
16. Define isomorphism and construct isomorphisms between groups, rings and fields.
17. Cite the Peano Axioms and explain why each is necessary to characterize the natural numbers.
18. Construct the integers from equivalence classes of ordered pairs of natural numbers.
19. Construct the set of rational numbers for equivalence classes of ordered pairs of integers.
20. Illustrate subsets of the rational numbers which are groups, rings and fields.

Computer Mathematics

Upon completion of the course, the student should be able to do the following performance objectives:

1. Display a knowledge of the history of computer science and the role of the computer in modern society.
2. Do program analysis of selected problems by constructing flow charts.
3. Interpret computer print-outs with special attention given to order, sequence and detail.
4. Design algorithms for selected problems, and write a program in a computer language to solve the problem.
5. Operate a computer keyboard or a time-share terminal as a means of computer input.
6. Develop a proficiency in some compiler language, such as BASIC or FORTRAN, in order to write and prepare programs to solve a variety of mathematical and scientific problems.
7. Rewrite (debug) programs which are rejected by the computer.
8. Operate related equipment, such as the teletypewriter, cardreader, printer, etc., necessary to process programs.
9. Discriminate among the results of the output, and choose what is significant.
10. Exhibit a knowledge of the career opportunities that exist in the computer science field.
11. List the limitations and strengths of the computer by noting what it can and cannot do.
12. Write increasingly complex programs for problems pertaining to other courses.

Calculus

Upon completion of the course, the student should be able to do the following performance objectives:

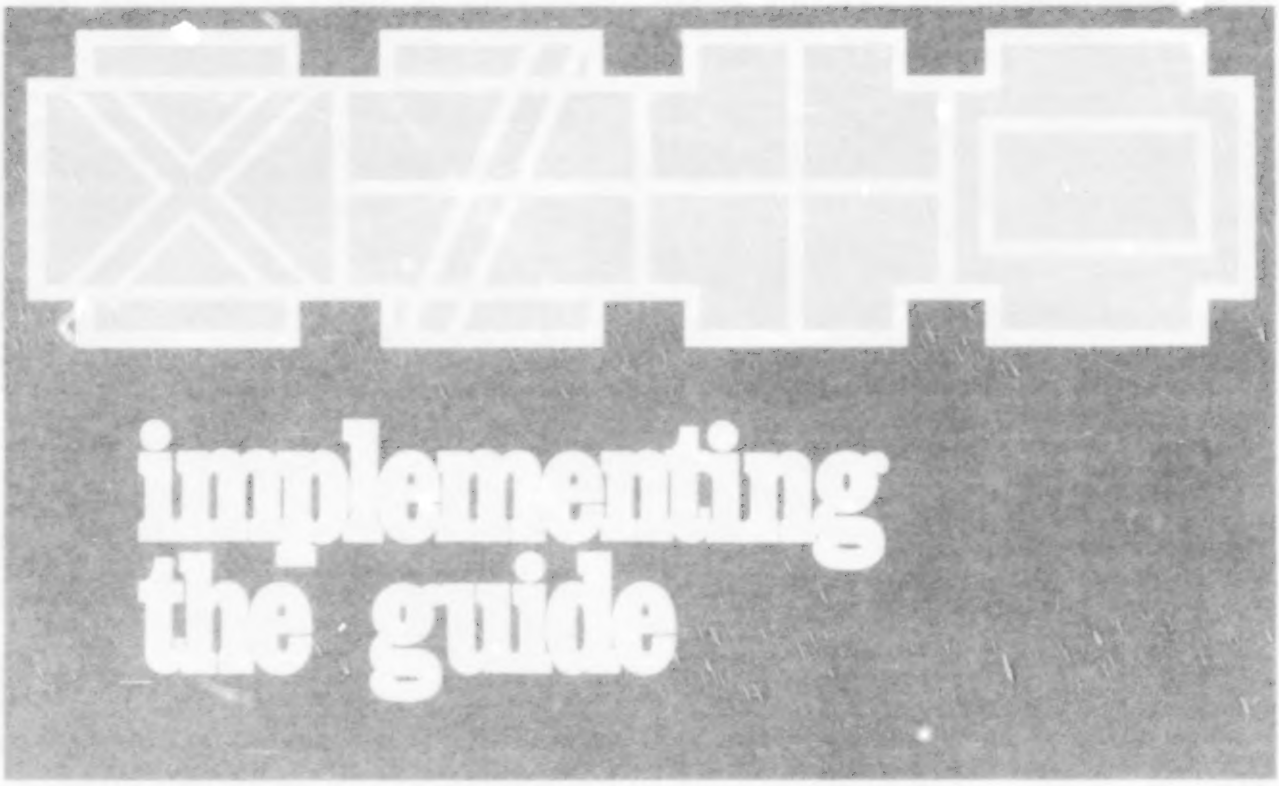
1. Define function, and state the domain and range of a function. Illustrate various methods of combining functions.
2. Completely analyze the properties of the conic sections, write their general equations using the definitions or standard forms, and write basic geometric proofs using analytic geometry.
3. Write the equation of a line using the two-point form, point-slope form, intercept form, or slope-intercept form; and express in the standard form, $Ax + By + C = 0$.
4. Graph functions and relations using the concepts of symmetry, asymptotes, domain, range, translation of axes, and rotation of axes.
5. Find the angle between two intersecting lines, and find the distance from a point to a line.
6. Find the limit at a finite point and at infinity, if either or both exist, of selected algebraic functions using the basic definition of limit.
7. Use L'Hospital's Rule, if applicable, to find the limit, if it exists, of a function which is discontinuous at a point.
8. Apply theorems on limits to find the limit at a finite point or at infinity, if either or both exist, of selected functions.
9. Prove $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$
10. Use the definition of a derivative to find the derivative of selected algebraic functions.
11. Use the sum, difference, product, quotient, and chain rule to find the derivative of algebraic, trigonometric, inverse trigonometric, exponential, logarithmic, and (hyperbolic functions).*
12. Use the derivatives of $f(x) = \sin x$ and $g(x) = \cos x$ to derive the derivatives of sec x, csc x, tan x, and cot x.
13. Use the derivative to find the extreme points, the points of inflection and the absolute maximum and minimum values of a function.
14. Use the derivative to find the equation of the tangent and normal line to a curve at a point on the curve.
15. Use the derivative to find the equation of the tangent lines from an external point to a curve.
16. Use the derivative to find velocity, acceleration, and solve related rate problems.
17. Use the derivative to find the angle between the graphs of two curves.
18. Use the derivative to approximate roots of an equation.
19. Use the derivative to determine whether or not a function is increasing or decreasing on a given interval.
20. Use the methods of substitution, partial fractions, and integration by parts to find the integral of algebraic, trigonometric, inverse trigonometric, exponential, logarithmic, and (hyperbolic functions).*
21. Use the definite integral to find area under a curve and between two curves, length of an arc*, surfaces of revolution*, and centroids*.
22. Use the definite integral to find volumes of solids of revolution, find volume by the cylindrical shell and disk methods, and find volume by the method of slicing.
23. Apply the integral to selected problems of physics such as hydrostatic force, work, acceleration and velocity.
24. Use the trapezoidal rule to approximate selected definite integrals.
25. Evaluate integrals which are infinite or discontinuous.
26. State and illustrate the basic theorems such as the fundamental theorems of differential and integral

*Optional

calculus and theorems on continuous functions.

27. Use the definite integral to compute centers of mass of objects with constant or variable densities.
28. Find and apply the derivative of vector functions.
29. Evaluate selected definite integrals using Simpson's rule.
30. Find and apply the derivative of equations written in polar coordinate form.
31. Prove the theorems stated and implied in number 26.*
32. Define the limit of a sequence and apply the theorems on the limits of sums, products and quotients of sequences.*
33. Use the alternating series test, the ratio test, the integral test and the comparison test.*
34. Find the Taylor's series for selected elementary functions.*
35. Determine the interval of convergence of power series and use the methods of term by term integration and differentiation to find power series of selected functions.*

*Optional



Implementation of the Guide

It was the intention of the advisory council to produce a document that would be of use to all segments of the educational community, students, teachers and administrators. Although the guide is designed neither to be diagnostic nor prescriptive, it is constructed with the belief that the learning process is enhanced by both the students and teachers having a knowledge of what is to be taught and learned.

The guide was designed for maximum usage particularly by classroom teachers located in school systems that do not have locally developed mathematics curriculum guides. However, the primary purpose of this guide is to stimulate local mathematics committees in the development of their own curriculum guides which should contain specific instructional or enabling objectives that include conditions and criteria for acceptable performance. These are the type of objectives that are necessary so that individual students may achieve mastery of the partial list of global objectives contained in this guide.

The initial phase in the development of a local mathematics curriculum based on these objectives or on those prepared locally should be identification of appropriate instructional objectives including conditions and criteria for acceptable performance. After these objectives have been identified, the succeeding task will be the selection of instructional materials that will aid the students in their attainment of the previously mentioned objectives. This guide was produced to help facilitate this sequence of events rather than the reverse order which has been too frequently practiced in the past.

Through the Division of Curriculum, Indiana Department of Public Instruction, the superintendent is willing to provide assistance to any school system in the development of local mathematics programs and guidelines. As a means to this end individual schools or corporations should feel free to duplicate any part of this guide for their own use. △

Materials Selection

The selection of materials to aid in the instruction of mathematics is one of the more important activities in which the teacher participates. Since the adoption of textbooks for a particular program or course normally occurs only once in a five-year period, great care and caution should be exercised when completing this task.

The Commission on Textbook Adoptions selects up to seven programs to be used in the instruction of a particular course that has been approved by the Commission on General Education. After the process

has been completed, the local school corporation reviews the state adopted materials to find those which are most suitable for use by the students in their school system.

If a school system wishes to use a textbook or program in lieu of those on the state adopted list, the corporation must secure a waiver from the Commission on Textbook Adoptions. It is the responsibility of the school system requesting the waiver to assure that each member of the Commission receives a copy of the proposed text for review. This waiver request should include the following:

1. Name and address of school corporation requesting the waiver.
2. Title, edition and publisher of the proposed text.
3. Indication of the local school officials endorsing the waiver request.
4. A rationale explaining why the proposed text is preferred over those on the state adopted list.
5. The proposed duration of the waiver, including the beginning and ending dates.
6. Number of classes or classrooms in which the text will be used.
7. Approximate number of pupils who will use the textbook.
8. Name of the school(s) in which the text will be used.
9. Signature of the local school official requesting the waiver.

The approval is granted for a period of one year with option for renewal contingent on filing an evaluation of the course or program with the commission.

If a school corporation wishes to offer a course that is not listed by the Commission on General Education, it may do so only upon the commission's approval after having filed a request. This request should include but not be limited to the following information:

Rationale — A statement of purpose for establishing the experimental course including the proposed title and how the needs of students will be served more effectively.

The Course and the School — The amount of credit to be awarded, whether the course is required or elective, whether the course will supplement or supplant other offerings, the characteristics of students the course is designed to serve, prerequisites, how the course organization will meet minimum standards as outlined by the State Board of Education, etc.

Course of Study and Student Objectives — What student growth should result from the intended instructional objectives, an outline of concepts to be developed, a listing of topics to be covered, and a sequential plan of course procedure.

Instructional Materials — Basic texts, if any, supplementary materials and equipment unique to instructional strategies.

Evaluation — A method, conceived prior to initiation of course and based upon course objectives, of determining the extent to which objectives of the course have been reached and student needs met.

Finance — How local, state and federal funds will be used to support the course.

General Information — Schools, addresses and principals involved, number of students, grade placement, ages, and criteria used for selection. Include here whether and how community resources are being used. Indicate the plan for information dissemination and visitation.

Planning — Steps taken, community and staff involvement.

Time limits and provisions for renewal of this waiver are the same as those stated in the proceeding paragraph.

After the state adoption has been completed, local selection committees should begin review of materials to find those that best fulfill the objectives of the school system and the needs of the students. If the local corporation does not have curriculum guides or course objectives, its adoption committee could duplicate and use the performance objectives included in previous sections of this guide. Also included is a copy of a textbook evaluation which may be used, with or without modifications, to aid in the selection of materials to be used in the instruction of mathematics.△

Sample Textbook Evaluation Form

1974 MATHEMATICS TEXTBOOK ADOPTION EVALUATION FORM

CATEGORY: Grade Level _____

Subject _____

TITLE OF BOOK(S) _____

_____

_____ COPYRIGHT DATE _____ 19 ____

AUTHOR(S) _____

_____

PUBLISHER _____

EVALUATOR _____ DATE _____ 19 ____

DIRECTIONS. Read each question slowly and thoroughly. It is important that you read them in their entirety.

To answer a question, mark an 'X', on the scale 0 to 6, to the left of the question between the hash marks over the number indicating the degree or extent you evaluate the material's characteristics.

In addition, you have been given a scale to the right of the question. This scale allows you to grade the value of the question. Please mark an 'X' along the side of the word which indicates the value you would apply to the question.

Finally, after you have marked the above two scales, you may further explain your answer by placing an 'X' alongside the words, "See Narrative." At the end of the form is space for written, subjective answers to clarify or further delineate your response to the question.

/ / / / X / / / /
0 1 2 3 4 5 6

No or little extent — To some extent — Great extent

___ _____ See Narrative

1. **To what extent does the title indicate the content of the book?**

Question is:
- ___ 0. Unimportant
- _X_ 1. Relatively Unimportant
- ___ 2. Moderately Important
- ___ 3. Important
- ___ 4. Extremely Important

sample

/ / / / / / / /
0 1 2 3 4 5 6

No or little extent — To some extent — Great extent

_____ See Narrative

1. **To what extent do the materials focus on the major concepts and/or big ideas pertinent to the mathematics category being evaluated?**

Question is:
- ___ 0. Unimportant
- ___ 1. Relatively Unimportant
- ___ 2. Moderately Important
- ___ 3. Important
- ___ 4. Extremely Important

/ / / / / / / /
0 1 2 3 4 5 6

No or little extent — To some extent — Great extent

_____ See Narrative

2. To what extent is the subject matter geared to the interests, abilities, and needs of the students for which the materials are being considered?

Question is:
___ 0. Unimportant
___ 1. Relatively Unimportant
___ 2. Moderately Important
___ 3. Important
___ 4. Extremely Important

/ / / / / / / /
0 1 2 3 4 5 6

No or little extent — To some extent — Great extent

_____ See Narrative

3. To what extent can the teacher depart from the sequence of materials prescribed by the author without impairing the effectiveness of the materials?

Question is:
___ 0. Unimportant
___ 1. Relatively Unimportant
___ 2. Moderately Important
___ 3. Important
___ 4. Extremely Important

/ / / / / / / /
0 1 2 3 4 5 6

No or little extent — To some extent — Great extent

_____ See Narrative

4. To what extent does the physical and technical (not substantive) quality of the student materials, including all media, meet professional expectations?

Question is:
___ 0. Unimportant
___ 1. Relatively Unimportant
___ 2. Moderately Important
___ 3. Important
___ 4. Extremely Important

/ / / / / / / /
0 1 2 3 4 5 6

No or little extent — To some extent — Great extent

_____ See Narrative

5. To what extent does the physical and technical quality of the supportive teacher materials (teacher's guide, media items, etc.) meet professional expectations?

Question is:
___ 0. Unimportant
___ 1. Relatively Unimportant
___ 2. Moderately Important
___ 3. Important
___ 4. Extremely Important

/ / / / / / / /
0 1 2 3 4 5 6

No or little extent — To some extent — Great extent

_____ See Narrative

6. To what extent does the language of the textbook accommodate the range of abilities of the students most likely to be using the textbook?

Question is:
___ 0. Unimportant
___ 1. Relatively Unimportant
___ 2. Moderately Important
___ 3. Important
___ 4. Extremely Important

/___/___/___/___/___/___/___/
0 1 2 3 4 5 6

No or little extent — To some extent — Great extent

_____ See Narrative

7. To what extent do the materials accommodate the range of reading abilities of the students most likely to be using the materials?

Question is:
____ 0. Unimportant
____ 1. Relatively Unimportant
____ 2. Moderately Important
____ 3. Important
____ 4. Extremely Important

/___/___/___/___/___/___/___/
0 1 2 3 4 5 6

No or little extent — To some extent — Great extent

_____ See Narrative

8. To what extent does the text use consistent, acceptable terminology and symbols?

Question is:
____ 0. Unimportant
____ 1. Relatively Unimportant
____ 2. Moderately Important
____ 3. Important
____ 4. Extremely Important

/___/___/___/___/___/___/___/
0 1 2 3 4 5 6

No or little extent — To some extent — Great extent

_____ See Narrative

9. To what extent does the material follow a logical, sequential development?

Question is:
____ 0. Unimportant
____ 1. Relatively Unimportant
____ 2. Moderately Important
____ 3. Important
____ 4. Extremely Important

/___/___/___/___/___/___/___/
0 1 2 3 4 5 6

No or little extent — To some extent — Great extent

_____ See Narrative

10. To what extent are fundamental principles correctly used throughout the text?

Question is:
____ 0. Unimportant
____ 1. Relatively Unimportant
____ 2. Moderately Important
____ 3. Important
____ 4. Extremely Important

/___/___/___/___/___/___/___/
0 1 2 3 4 5 6

No or little extent — To some extent — Great extent

_____ See Narrative

11. To what extent does the text develop concepts and skills in a spiral manner?

Question is:
____ 0. Unimportant
____ 1. Relatively Unimportant
____ 2. Moderately Important
____ 3. Important
____ 4. Extremely Important

/ / / / / / / /
0 1 2 3 4 5 6

No or little extent — To some extent — Great extent

_____ See Narrative

12. **To what extent does the text provide the student with an opportunity to develop both inductive and deductive reasoning?**

Question is:
____ 0. Unimportant
____ 1. Relatively Unimportant
____ 2. Moderately Important
____ 3. Important
____ 4. Extremely Important

/ / / / / / / /
0 1 2 3 4 5 6

No or little extent — To some extent — Great extent

_____ See Narrative

13. **To what extent are definitions and explanatory materials clear and precise?**

Question is:
____ 0. Unimportant
____ 1. Relatively Unimportant
____ 2. Moderately Important
____ 3. Important
____ 4. Extremely Important

/ / / / / / / /
0 1 2 3 4 5 6

No or little extent — To some extent — Great extent

_____ See Narrative

14. **To what extent does the text present exercises which are adequate to develop the pupil's mastery of the concept being presented?**

Question is:
____ 0. Unimportant
____ 1. Relatively Unimportant
____ 2. Moderately Important
____ 3. Important
____ 4. Extremely Important

/ / / / / / / /
0 1 2 3 4 5 6

No or little extent — To some extent — Great extent

_____ See Narrative

15. **To what extent does the content of the textbook — both the pictorial and written content — reflect the pluralistic, multi-ethnic nature of our society, both past and present?**

Question is:
____ 0. Unimportant
____ 1. Relatively Unimportant
____ 2. Moderately Important
____ 3. Important
____ 4. Extremely Important

/ / / / / / / /
0 1 2 3 4 5 6

No or little extent — To some extent — Great extent

_____ See Narrative

16. **To what extent does the material depict both male and female members of various groups of society in situations which exhibit them unbiasedly?**

Question is:
____ 0. Unimportant
____ 1. Relatively Unimportant
____ 2. Moderately Important
____ 3. Important
____ 4. Extremely Important

/ / / / / / / /
0 1 2 3 4 5 6

No or little extent — To some extent — Great extent

17. To what extent would the book tend to encourage a positive self-image for children of all ethnic and socio-economic groups?

Question is:
____ 0. Unimportant
____ 1. Relatively Unimportant
____ 2. Moderately Important
____ 3. Important
____ 4. Extremely Important

_____ See Narrative

/ / / / / / / / /
0 1 2 3 4 5 6

No or little extent — To some extent — Great extent

18. To what extent does the book develop accurate mathematical concepts and generalizations?

Question is:
____ 0. Unimportant
____ 1. Relatively Unimportant
____ 2. Moderately Important
____ 3. Important
____ 4. Extremely Important

_____ See Narrative

/ / / / / / / / /
0 1 2 3 4 5 6

No or little extent — To some extent — Great extent

19. To what extent does the author stress the importance of rational thought in discovering and testing the validity of concepts?

Question is:
____ 0. Unimportant
____ 1. Relatively Unimportant
____ 2. Moderately Important
____ 3. Important
____ 4. Extremely Important

_____ See Narrative

/ / / / / / / / /
0 1 2 3 4 5 6

No or little extent — To some extent — Great extent

20. To what extent are there opportunities for alternative teaching strategies?

Question is:
____ 0. Unimportant
____ 1. Relatively Unimportant
____ 2. Moderately Important
____ 3. Important
____ 4. Extremely Important

_____ See Narrative

/ / / / / / / / /
0 1 2 3 4 5 6

No or little extent — To some extent — Great extent

21. To what extent would the materials contained in this textbook be learnable and/or teachable?

Question is:
____ 0. Unimportant
____ 1. Relatively Unimportant
____ 2. Moderately Important
____ 3. Important
____ 4. Extremely Important

_____ See Narrative

/ / / / / / / /
0 1 2 3 4 5 6

No or little extent — To some extent — Great extent

_____ See Narrative

22. To what extent would you (the reviewer) recommend that these materials be used? *Note:* Question number 22 receives three (3) times the importance of any single preceding question.

Question is:
____ 0. Unimportant
____ 1. Relatively Unimportant
____ 2. Moderately Important
____ 3. Important
____ 4. Extremely Important

/ / /
Yes No

23. Should this textbook material be one (1) of the textbooks adopted in this category?

/ / / / / / / /
1st 2nd 3rd 4th 5th 6th 7th
Choice Choice

24. If your answer for question number 23 was yes, how would you rank this textbook?

/ / /
Yes No

25. Are these materials designed for students with reading disabilities? *Note*: This question is only for textbook reviewers of materials in the two divergent abilities categories.

/ / / / / / / /
0 1 2 3 4 5 6

No or little extent — To some extent — Great extent

_____ See Narrative

26. To what extent does the title indicate the content of the book?

Question is:
____ 0. Unimportant
____ 1. Relatively Unimportant
____ 2. Moderately Important
____ 3. Important
____ 4. Extremely Important

NARRATIVE SECTION

Question number _____

Question number _____

Question number _____

Question number _____

Question number _____

Question number _____

Question number _____

Question number _____

Mathematics Curriculum Development Process

Step I — Selection of Mathematics Curriculum Committee

1. One teacher from each grade level

a) An outstanding teacher in the field of mathematics. b) Experienced and enthusiastic but not domineering. c) A teacher with respect and expertise to obtain valuable written opinions of non-member colleagues. d) A committee is no better than the selection.

2. Appoint a chairperson

a) One who is knowledgeable in the field of mathematics. b) A statesman who is objective and fair, not a politician. c) A stable person who most likely will return next year.

3. Appoint lay individuals.

a) Individuals willing and able to be of value. b) Individuals whose ideals are respected by the majority of your corporation populace. c) Individuals willing to make a commitment and give reason.

Note — Committee can get too large and therefore should not exceed 12 members.

Step II — The committee compiles a mathematics survey summary related to examples A and B from a questionnaire sent to and collected from all teachers of mathematics, shop, home economics, business, high school mathematics faculty and others.

Example A. According to a survey of _____ teachers with mathematics teaching responsibility, the following are seen as problem areas in the K-8 mathematics program. More frequently listed concerns are listed below: 1) Supplementary materials are in short supply. 2) Insufficient drill on basic facts and concepts. 3) Slower readers lack appropriate materials. 4) Acceptable degree of mastery is not defined or provided. 5) Lack of standard of evaluation.

After compiling the information concerning the identified problem areas, the committee should do the following:

1. Committee isolates mentioned problem areas for later consideration which are not mathematics program problems but individual teacher problems.
2. Committee selects from the mentioned problem areas weaknesses of the presently used mathematics program that will not be substantially aided by the best possible single textbook.
3. By a majority agreement, the committee selects and gives a priority rating to *x* identified "problem areas" it wishes to address initially.
4. Committee makes a study of "characteristics of a good mathematics program." The opinions of colleagues can be found by means of a questionnaire such as Example B:

Example B. Teachers suggested the following as characteristics of a good mathematics program:

1. Should provide for individual differences.
2. Math games and concrete materials should be used freely when appropriate.
3. Math facts should be memorized through adequate drill.
4. A skills mastery checklist should be used to record student progress.
5. The program should be systematic and continuous.

After gathering the data on the characteristics of a good program, the committee:

1. Identifies and temporarily ignores those characteristics which are strengths of the present program.
2. Selects from the remaining "characteristics of a good mathematics program" *x* items and gives them an order of priority.
3. Reviews "problem area" priorities and examines possible correlation to the chosen high priority characteristics of "a good mathematics program."
4. If the correlation of the two sets of priorities is unacceptable to the committee, the activities of Step II should be repeated before proceeding to Step III.

Step III — The committee proceeds to draft a philosophy reflecting local conditions as well as proven methodologies. This process consists of the following:

1. Determining the areas for consideration.
2. Defining these areas.
3. Writing a philosophy based on these definitions.

Example considerations: environment, history, language, application, quality, support and appreciation.

Definitions of Considerations. Environment — instead of expecting all children at a given grade level to master (in a rather final sense) a certain portion of mathematical training, educators should develop an approach which recognizes the existence of individual growth rates and stresses the continuity of the instructional process.

History — it is no longer deemed sufficient for a teacher to be satisfied with competent computational performance by a child in spite of the obvious necessity for such skills. The need to find more efficient, more enjoyable, more illuminating methods of instruction has led to the consensus that clear and penetrating understanding of certain essential mathematics must precede, but certainly not supplant computation. Clearly, children should be encouraged to develop their first concepts of mathematics from experience with physical objects.

Language — teachers at each level should ascertain that the instruction is directed toward deepening and extending the broad mathematical streams. As a child's grasp of mathematics grows, he must be guided toward the acquisition of the broad and essential concepts which tie together seemingly disconnected particulars into a coherent, general structure. With the aid of consistency, children must be taught to interpret symbols in context. Only in this way will mathematics shorthand serve the important purpose of clarifying

and improving communication.

Application — children should come to realize that mathematics is a logical system which exists as a human invention; it is formulated, enriched, extended and revised in response to the twin needs to perfect it as a logical structure and to use it as a convenient method of describing certain aspects of nature as seen by man.

Quality — educators should avoid giving undue emphasis to any single aspect of the total mathematics program. The idea of sets, for instance, should not be glorified beyond its usefulness in contributing to the attainment of the broad goals of the program. Similarly, it is unwise to go to the extreme of downgrading the importance of computational proficiency to the point where long range goals are placed in jeopardy. Furthermore, impressive terms and symbols must never interfere with a child's understanding of the concepts which the words or symbols represent.

Support — many word problems fail to challenge children; they fail to represent a world which is real to children; and too often they fail to contribute to the child's skill in problem-solving situations outside the textbook. There is increasing evidence that through experimental programs and through the efforts of individual classroom teachers, improved ways of meeting this obligation are being developed. Much is being heard about units of study in mathematics as it relates to ecology and other areas of enrichment activities in mathematics, about the study of the history of mathematics, and about children creating their own mathematical problems.

Appreciation — efforts are being made to emphasize the structure of mathematics through problem-solving experiences. These activities are hopeful signs pointing to a greater stress being placed on the importance of meaningful, structured problem-solving experiences as a part of the elementary mathematics program.

Philosophy. Language — Mathematics instruction should be viewed as a continuous effort to develop in the child a knowledge that is characterized by its depth and connectedness.

Environment, History — to the extent possible, the child should be encouraged to experiment with the objects of this environment.

Application, Support — thus prepared, the child may be led to the invention or discovery of those ideas which provide both a broad basis for further exploration and a sense of delight in a well-founded mastery of the subject.

Quality, Support — the task is a challenging one and deserves much effort. To this end, the teacher should do everything possible to see that instruction is well planned and is provided on a regular basis.

Appreciation — as an additional but essential measure, cooperative action such as inservice education should be taken by school personnel to develop in the teaching staff a view of instruction appropriate to present day needs and opportunities.

Step IV — (Optional) The committee may choose to write related global objectives. Examples:

1. Environment — the student should come to comprehend how mathematics contributes to the analysis of events that occur in the physical world.
2. History — the student needs to be aware of the important role mathematics plays in the evolution of civilization.
3. Language — the student needs to use the words and symbols of mathematics with precision so that mathematical ideas can be communicated correctly and clearly to others.
4. Appreciation — students need to develop a mathematical literacy which will enable them to make wise decisions as a producer and/or consumer of products and services.
5. Quality — the student needs to acquire proficiency in the use of common mathematical processes.
6. Support — the student needs to become sufficiently prepared at each level for further study of mathematical concepts and operations.
7. Appreciation — the student needs to acquire habits of orderliness, competence in logical argument, and an appreciation of the nature of mathematics as a logical, deductive system built on a structure of unifying concepts.

Step V — With the assistance of colleagues, each committee member compiles a comprehensive grade level set of instructional objectives which are consistent with the committee's adopted philosophy. Methods for generating appropriate objectives include:

1. Writing objectives by grade level, making sure to have some overlap.
2. Delegating some of these responsibilities to non-committee members, but being careful to explain philosophy.
3. Choosing previously written objectives whenever possible. (This guide is a source of such objectives.)

Step VI — The committee acts as a whole to prepare and edit the final list of objectives.

Step VII — The committee should provide examples of activities which are related to the objectives.

Step VIII — The committee should make decisions concerning:

1. Examples of instructional materials which would complement and enrich the adopted philosophy and related objectives.
2. Types of inservice needed.
3. Additional supplemental supplies needed.
4. Types of administrative support needed to complement the program.
5. Community resources available for use.

Step IX — Provision for both summative and formative evaluation procedures will be prepared by the committee. Δ

Role of Schools in Metric Conversion

Note: This position paper, prepared by the Metric Education Task Force and funded through ESEA Title III, was first published by the Indiana Department of Public Instruction in February, 1976.

This comprehensive position statement reflects the many hours of thought and discussion the Indiana Metric Education Task Force has devoted to dealing with the problems of metric conversion for our school programs.

In their statement, the task force urges each of us to get on with the task of presenting measurement in metric terms. Let me reinforce their message with an assurance that the Department of Public Instruction will vigorously increase its efforts to assist all Indiana school corporations with this important enterprise.

Harold H. Negley,
Superintendent

Introduction

Recent events clearly indicate that the United States is moving rapidly toward a functional, if not formal, adoption of the metric measurement system. Although use of the metric system of weights and measures became legal in 1866 and conventional measurement units were officially defined in terms of metric standards in 1893, the United States continues to utilize the American version of the British System of Measurement. England officially adopted the metric system in 1965 and nearly all English-speaking nations are now implementing the metric system; consequently, American business and industry are having to adopt metric dimensioning in order to speak the same language as their competitors. Obsolete product designs and machinery are being replaced with those using metric dimensions. Additionally, all government contracts require metric labeling. These facts make it apparent that full conversion to metric measurement is inevitable.

Since one generally accepted goal of schools is preparing students to participate in tomorrow's world, schools are obligated to prepare them for easy adjustment to the measurement system which they will use throughout most of their adult lives. The importance of metric instruction in the schools is further stressed by the Education Act of 1974, Public Law 93-380, Section 403a.2: "It is the policy of the United States to encourage educational agencies and institutions to prepare students to use the metric system of measurement with ease and facility as a part of the regular education program."

Requests from schools across the state indicate an immediate need for programs to provide teachers with the needed background, confidence and inspiration for classroom implementation of metric instruction. An abundance of materials is now available, and many suggestions have been advanced for teaching the metric system. However, teachers have little assurance that these materials and suggestions are based on sound learning theory, that they are designed to fulfill the goals of their curriculum, or that they are in concert with efforts underway in other segments of society. To meet these needs and to aid the Indiana Department of Public Instruction in developing a consistent educational direction and support system, Harold H. Negley, superintendent, Indiana Department of Public Instruction, appointed a task force to function through the ESEA Title III funded Science and Mathematics Curriculum Exploration Center at Jeffersonville. The task force was charged with developing a position statement on the role of Indiana schools in the conversion to the metric system.

This position statement enumerates various activities underway in Indiana, identifies activities which should be expanded or initiated, delineates those activities which should be undertaken by the schools, suggests support systems needed for the schools' activities and proposes guiding principles which should underlie all activities related to metric conversion.

Activities Underway in Indiana

Students in Indiana schools receive some instruction in metric measurement in their mathematics, science and practical arts classes. Teachers are provided inservice and preservice programs by a variety of sponsoring organizations, including professional organizations, institutions of higher education, ESEA Title III projects, local school corporations and various divisions of the Indiana Department of Public Instruction.

Adults are offered metric education through a variety of continuing education programs. Employees of businesses and industries within Indiana which are committed to metric conversion are being given appropriate instruction.

Some governmental agencies, through field service operations, furnish personnel and materials for the education of the general public. Some businesses and industries, as well as the Indiana State Highway Commission, increase public awareness of the metric system through dual labeling. Various parent-community-school organizations (PTA, PTO, PTSA, etc.) are presenting metric awareness programs. The mass media also contribute to public awareness of metric measurement on an occasional basis.

Activities Which Need to Be Expanded or Initiated

Businesses, industries and governmental agencies involved in metric conversion should intensify efforts to establish the metric measurement habit among employees as well as to alert the general public to the need for conversion. All agencies concerned with public information and education should increase their support of continuing education programs fostering metric awareness and utilization. Guidelines and materials should be developed and/or disseminated for use in conducting public discussions on metrication.

Federal leadership is needed to coordinate installation of the metric system as the responsibility for conversion will be spread among all sectors of society. This leadership is needed to facilitate full metric

conversion by business and industry so that competition for the domestic market does not stifle voluntary conversion.

The mass media (television, radio and newspapers) would seem to offer the most pervasive influence in the general conversion process. Television, for example, can devote regular news coverage or special programs to metrication. When reporting news stories which use measurements, that information can be displayed on the screen in both metric and British units. The vidifont will allow reinforcement without time required for verbal explanation.

Metric units should be used as the primary measurement system for reporting information. For example, sporting events in nations using the metric system should be reported in native units; conventional units should appear only as parenthetical data for comparison with American records. Further, it is recommended that reports of American events provide conversions to metric dimensions.

Service agencies which furnish weather information to the public can encourage acceptance of the metric system by always reporting the Celsius temperature. These agencies should develop a schedule to begin reporting all weather data in metric units as well as comparative conventional units. This reporting includes time and temperature displays and "dial the weather" services in addition to weather reporting and forecasting through the mass media.

Programs to educate the school age population should be expanded so every student can acquire a metric measurement habit. To acquire this habit students must utilize metric units in all areas of the curriculum. Therefore, all teachers must encourage the continual use and application of metric measuring. Although teachers entering the profession will gain metric concepts and methods during preservice programs, it will take time for these teachers to have general impact on the curriculum. Therefore, schools, state agencies, institutions of higher education and professional education organizations must immediately intensify efforts to provide practicing teachers and school administrators appropriate inservice programs. These programs must provide for rapid dissemination of metric concepts and methods to all teachers. One method of dissemination would be training a teacher in each school or school corporation to provide inservice programs for other teachers.

Activities Which Should Be Undertaken by Schools

The influence of schools in metric conversion falls into three categories: 1) developing a metric measurement habit among students, 2) supplying portions of the general public awareness program and 3) providing counsel to other segments of society in developing training programs.

For students to develop a metric measurement habit, metrication instruction should extend into all curriculum areas and continue throughout the students' educational careers. Existing curriculum guides should be revised to include metric concepts for implementation through updated lesson plans.

Mathematics instruction should use metric units in activities which develop measurement concepts and competencies. Science classes have long employed metric measurement and this tradition should not only be continued but be enlarged upon. Practical arts programs should require that students use tools and other equipment calibrated in metric units. (For the purposes of this paper, the term "practical arts" includes but is not limited to trade and industrial education, agribusiness, vocational training, home economics, business and distributive education.) Social sciences can help students explore the economic complexities mandating conversion to metrics. The humanities can support conversion by examining the benefits of common terminology in international communications.

Schools should revise specifications used in selecting and purchasing instructional materials, supplies and equipment. Teaching aids and manipulative devices used in developing measurement concepts should be either calibrated solely in metric units or both metric and conventional units. Special attention should be given to the specifications for new equipment and tools for practical arts classes, i.e., require metric dimensions and calibrations.

The responsibility for developing a metric measurement habit among students ultimately rests with teachers; therefore, coordinated inservice and preservice programs are needed to prepare teachers to use consistent terms and standards. Since the metric system should be incorporated into all aspects of school life, all teachers as well as administrators must be provided experiences from which they can internalize those metric frames of reference needed to intelligently select metric materials consistent with the overall school goals and objectives.

The schools can support general public awareness through a variety of continuing education programs, including cooperative activities with parent-community-school organizations. These programs are necessary not only to foster public support for metric conversion, but to aid in developing a general understanding of the schools' activities in these efforts.

Educational personnel experienced in implementing metrication programs can, and should, provide counsel to other sectors of society as they retool for metrication. This counsel should include, but not be limited to, recommending appropriate training materials and methodology.

How Can Other Segments of Society Support the Schools?

As schools strive to fulfill their role in conversion to the metric system, they will need support and aid from other sectors of society. Many supportive activities have already been suggested. For instance, since education of children must relate to their real world experiences, the assistance the mass media provide to public awareness supports the schools' efforts in metrication.

Professional organizations can support the schools with inservice training and awareness programs. These organizations can organize resources to aid in the

development of materials based on sound learning theory and promote public support of the schools' metrication efforts.

Materials developed for employee training programs should be made available for school use when appropriate. Further, businesses and industries directly involved in metric conversion should provide opportunities for students, and particularly teachers, to understand the complexity of the problems resulting from the transition.

Agencies sponsoring competitive events for school age participants should convert to metric dimensions and report performance records in metric units.

Finally, there must be federal and state funds dedicated to developing and disseminating metric objectives, curriculum guides, resource directories, activities and materials.

Guiding Principles

The use of common goals and terminology will allow for intrastate, interstate and international communication and mobility. The International Organization for Standardization (ISO), the specialized international agency for standardization representing 72 nations, has officially adopted the International System of Measurement (SI). More than 90 per cent of the world's population resides in countries officially using the SI system. The SI system has been adopted by the United States National Bureau of Standards and Public Law 93-380, Section 403 a.3. states that: ". . . the term 'metric system of measurement' means the International System of Units as established by the general conference of weights and measures in 1960 and interpreted or modified for the United States by the Secretary of Commerce."

Therefore, this task force recommends the following in regard to all metric efforts in Indiana:

1. Accept the SI system as the standard measurement system.
2. Alert students to the use of both the "er" and "re" spellings of length and capacity units. Although the "er" spelling may be accepted in everyday usage, "metre" and "litre" should be employed for all technical writings.
3. Accent metric prefixes in pronunciation, e.g. kil'o-metre, to help communicate the numerical concept expressed and to avoid confusion.
4. Develop the correct concepts of mass and force by using the "kilogram" as the unit of mass and the "newton" as the unit of force and weight. The task force recognizes that, in day-to-day conversation, measuring the matter present in objects may continue to be referred to as "weighing."
5. Treat the SI and British systems as independent systems in labeling and instruction to minimize confusion during the transition period.
6. Be consistent in the accuracy used in reporting parallel measurements, i.e., use the same number of significant digits in both systems.
7. Avoid conversion between the SI system and other systems; however, in situations where conversion is required, appropriate uniform conversion tables, charts, graphs, calculators, etc., should be made available.
8. Infuse metrication throughout the entire school curriculum; do not treat it as an isolated topic of study.
9. Begin preparing for classroom instruction on the SI system of measurement during the 1975-76 school year.

Summation

The major instructional responsibility for metric conversion rests with the educational community. Therefore, educators must cast aside reservations and encourage conversion to the metric system through practical applications with minimum confusion and chaos. However, the development of a functional metric habit among students must be strengthened and reinforced by experiences outside the formal school setting. The schools' role cannot be fulfilled without the coordinated, multilateral support of consumer, labor, professional, technical, trade, state, federal and civic organizations.

INDIANA METRIC EDUCATION TASK FORCE
Commissioned in 1975

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Inquiries regarding compliance with Title IX may be directed to the Division of Equal Educational Opportunity, Indiana Department of Public Instruction, 120 W. Market Street — 10th Floor, Indianapolis, IN, 46204, (317) 633-4978, or to the Director of the Office for Civil Rights, Department of Health, Education and Welfare, Washington, D.C. — **Harold H. Negley, State Superintendent of Public Instruction**